6.226.

$y = arccosx$

$y' = (arccosx)' = \frac{-1}{\sqrt{1-x^2}}$

$y'' = (\frac{-1}{\sqrt{1-x^2}})' = -1 \cdot [(1-x^2)^{-\frac{1}{2}}]' = -1 \cdot (-\frac{1}{2}) \cdot (1-x^2)^{-\frac{1}{2}-1} \cdot (1-x^2)' = \frac{1}{2} \cdot (1-x^2)^{-\frac{3}{2}} \cdot (0-2x) =$
$= -x \cdot \frac{1}{\sqrt{(1-x^2)^3}}$

6.227.

$y = arctg2x$

$y' = (arctg2x)' = \frac{1}{1+(2x)^2} \cdot (2x)' = \frac{2}{1+4x^2}$

$y'' = (\frac{2}{1+4x^2})' = 2 \cdot (1+4x^2)^{-1} = 2 \cdot (-1) \cdot (1+4x^2)^{-2} \cdot (1+4x^2)' = -2 \cdot \frac{1}{(1+4x^2)^2} \cdot (0+4 \cdot 2x) = -\frac{16x}{(1+4x^2)^2}$

6.228.

$y = (arcsinx)^2$

$y' = [(arcsinx)^2]' = 2 \cdot arcsinx \cdot (arcsinx)' = 2arcsinx \cdot \frac{1}{\sqrt{1-x^2}}$

$y'' = (2arcsinx \cdot \frac{1}{\sqrt{1-x^2}})' = [2arcsinx \cdot (1-x^2)^{-\frac{1}{2}}]' = (2arcsinx)' \cdot (1-x^2)^{-\frac{1}{2}} + (2arcsinx) \cdot [(1-x^2)^{-\frac{1}{2}}]' =$
$= 2 \cdot \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{\sqrt{1-x^2}} + 2arcsinx \cdot (-\frac{1}{2}) \cdot (1-x^2)^{-\frac{1}{2}-1} \cdot (1-x^2)' = \frac{2}{1-x^2} - arcsinx \cdot (1-x^2)^{-\frac{3}{2}} \cdot (0-2x) =$
$= \frac{2}{1-x^2} - arcsinx \cdot \frac{1}{\sqrt{(1-x^2)^3}} \cdot (-2x) = \frac{2}{\sqrt{(1-x^2)^2}} + \frac{2x \cdot arcsinx}{\sqrt{(1-x^2)^3}} = \frac{2\sqrt{1-x^2}}{\sqrt{(1-x^2)^3}} + \frac{2x \cdot arcsinx}{\sqrt{(1-x^2)^3}} = \frac{2(\sqrt{1-x^2}+x \cdot arcsinx)}{\sqrt{(1-x^2)^3}}$

6.229.

$y = ln(1+x^2)$

$y' = [ln(1+x^2)]' = \frac{1}{1+x^2} \cdot (1+x^2)' = \frac{1}{1+x^2} \cdot (0+2x) = \frac{2x}{1+x^2}$

$y'' = (\frac{2x}{1+x^2})' = \frac{(2x)' \cdot (1+x^2) - 2x \cdot (1+x^2)'}{(1+x^2)^2} = \frac{2 \cdot (1+x^2) - 2x \cdot (0+2x)}{(1+x^2)^2} = \frac{2+2x^2-4x^2}{(1+x^2)^2} = \frac{2-2x^2}{(1+x^2)^2} = \frac{2 \cdot (1-x^2)}{(1+x^2)^2}$

6.230.

$y = ln(\sqrt[3]{1+x^2}) = ln(1+x^2)^{\frac{1}{3}}$

Skorzystajmy z wzoru $log_a b^r = r \cdot log_a b$ i zapiszmy funkcję y następująco:

$y = \frac{1}{3} \cdot ln(1+x^2)$

Teraz skorzystajmy z wyniku zadania 6.229:

$y'' = \frac{1}{3} \cdot \frac{2\cdot(1-x^2)}{(1+x^2)^2} = \frac{2\cdot(1-x^2)}{3\cdot(1+x^2)^2}$

6.231.

$y = x \cdot e^{sinx}$

$y' = (x \cdot e^{sinx})' = x' \cdot e^{sinx} + x \cdot (e^{sinx})' = e^{sinx} + x \cdot e^{sinx} \cdot (sinx)' = e^{sinx} + x \cdot cosx \cdot e^{sinx} = e^{sinx}(1 + x \cdot cosx)$

$y'' = [e^{sinx}(1 + x \cdot cosx)]' = (e^{sinx})' \cdot (1 + xcosx) + e^{sinx} \cdot (1 + xcosx)' = e^{sinx} \cdot (sinx)' \cdot (1 + xcosx) +$

$+ e^{sinx} \cdot [0 + x' \cdot cosx + x \cdot (cosx)'] = e^{sinx} \cdot cosx \cdot (1 + x \cdot cosx) + e^{sinx} \cdot (cosx - x \cdot sinx) =$

$= e^{sinx}[cosx \cdot (1 + x \cdot cosx) + cosx - x \cdot sinx) = e^{sinx}(cosx + x \cdot cos^2x + cosx - x \cdot sinx) =$

$= e^{sinx}(2 \cdot cosx + x \cdot cos^2x - x \cdot sinx)$

6.232.

$y = e^{\varphi(x)}$

$y' = (e^{\varphi(x)})' = e^{\varphi(x)} \cdot \varphi'(x)$

$y' = [e^{\varphi(x)} \cdot \varphi'(x)]' = (e^{\varphi(x)})' \cdot \varphi'(x) + e^{\varphi(x)} \cdot \varphi''(x) = e^{\varphi(x)} \cdot \varphi'(x) \cdot \varphi'(x) + e^{\varphi(x)} \cdot \varphi''(x) =$

$= e^{\varphi(x)} \cdot [(\varphi'(x))^2 + \varphi''(x)]$

6.233.

$y = \sqrt[5]{x^3} = x^{\frac{3}{5}}, \qquad x > 0$

$y' = (x^{\frac{3}{5}})' = \frac{3}{5} \cdot x^{\frac{3}{5}-1} = \frac{3}{5} \cdot x^{-\frac{2}{5}}$

$y'' = (\frac{3}{5} \cdot x^{-\frac{2}{5}})' = \frac{3}{5} \cdot (-\frac{2}{5}) \cdot x^{-\frac{2}{5}-1} = -\frac{6}{25} \cdot x^{-\frac{7}{5}}$

$y''' = (-\frac{6}{25} \cdot x^{-\frac{7}{5}})' = -\frac{6}{25} \cdot (-\frac{7}{5}) \cdot x^{-\frac{7}{5}-1} = \frac{42}{125} \cdot x^{-\frac{12}{5}}$

6.234.

$y = \frac{1+x}{1-x}$ dla $x \neq 1$

$y' = (\frac{1+x}{1-x})' = \frac{(1+x)' \cdot (1-x) - (1+x) \cdot (1-x)'}{(1-x)^2} = \frac{1 \cdot (1-x) - (1+x) \cdot (-1)}{(1-x)^2} = \frac{1-x+1+x}{(1-x)^2} = \frac{2}{(1-x)^2}$

$y'' = (\frac{2}{(1-x)^2})' = 2 \cdot [(1-x)^{-2}]' = 2 \cdot (-2) \cdot (1-x)^{-3} \cdot (1-x)' = -4 \cdot (1-x)^{-3} \cdot (-1) = 4 \cdot (1-x)^{-3}$

$y''' = [4 \cdot (1-x)^{-3}]' = 4 \cdot (-3) \cdot (1-x)^{-4} \cdot (1-x)' = -12 \cdot (1-x)^{-4} \cdot (-1) = 12 \cdot (1-x)^{-4} = \frac{12}{(1-x)^4}$

6.235.

$y = sin(1-3x)$

$y' = [sin(1-3x)]' = cos(1-3x) \cdot (1-3x)' = -3 \cdot cos(1-3x) \qquad (1)$

$y'' = [-3 \cdot cos(1-3x)]' = -3 \cdot [cos(1-3x)]' = -3 \cdot [-sin(1-3x) \cdot (1-3x)'] = -3 \cdot 3 \cdot sin(1-3x) =$

$= -9 \cdot sin(1-3x)$

$y''' = [-9 \cdot sin(1-3x)]' = korzystamy\, z\, (1) = -9 \cdot (-3) \cdot cos(1-3x) = 27 \cdot cos(1-3x)$

6.236.

$y = arcsinx$ w punkcie $x = 0$

$y' = (arcsinx) = \frac{1}{\sqrt{1-x^2}}$

$y'' = (\frac{1}{\sqrt{1-x^2}})' = [(1-x^2)^{-\frac{1}{2}}]' = -\frac{1}{2} \cdot (1-x^2)^{-\frac{1}{2}-1} \cdot (1-x^2)' = -\frac{1}{2} \cdot (1-x^2)^{-\frac{3}{2}} \cdot (-2x) = \frac{x}{\sqrt{(1-x^2)^3}}$

Zatem:

$y''(0) = \frac{0}{\sqrt{(1-0^2)^3}} = 0$

6.237.

$y = \frac{x+2}{x^2-3x}$ w punkcie $x = 2$ $x^2 - 3x \neq 0$

$y = \frac{x+2}{x^2-3x} = \frac{x+2}{x(x-3)} = \frac{a}{x} + \frac{b}{x-3} = \frac{a(x-3)+bx}{x(x-3)} \Rightarrow a(x-3) + bx = x + 2$

$ax - 3a + bx = x + 2$

$(a+b)x - 3a = x + 2$

Zatem:

$$\begin{cases} a+b=1 \\ -3a=2 \end{cases}$$

$$\begin{cases} a+b=1 \\ a=-\frac{2}{3} \end{cases}$$

$-\frac{2}{3}+b=1 \Leftrightarrow b=1+\frac{2}{3} \Leftrightarrow b=\frac{5}{3}$

A więc naszą funkcję możemy zapisać następująco:

$y=\frac{-\frac{2}{3}}{x}+\frac{\frac{5}{3}}{x-3}=-\frac{2}{3}\cdot\frac{1}{x}+\frac{5}{3}\cdot\frac{1}{x-3}$

Teraz możemy łatwiej obliczyć kolejne pochodne:

$y'=(-\frac{2}{3}\cdot\frac{1}{x})'+(\frac{5}{3}\cdot\frac{1}{x-3})'=\frac{5}{3}\cdot[(x-3)^{-1}]'-\frac{2}{3}\cdot(x^{-1})'=\frac{5}{3}\cdot(-1)\cdot(x-3)^{-2}\cdot(x-3)'-\frac{2}{3}\cdot(-1)\cdot x^{-2}=$

$=\frac{2}{3}x^{-2}-\frac{5}{3}\cdot(x-3)^{-2}$

$y''=[\frac{2}{3}x^{-2}-\frac{5}{3}\cdot(x-3)^{-2}]'=(\frac{2}{3}x^{-2})'-[\frac{5}{3}\cdot(x-3)^{-2}]'=\frac{2}{3}\cdot(-2)\cdot x^{-3}-\frac{5}{3}\cdot(-2)\cdot(x-3)^{-3}\cdot(x-3)'=$

$=\frac{10}{3}\cdot\frac{1}{(x-3)^3}\cdot 1-\frac{4}{3}\cdot\frac{1}{x^3}$

Zatem:

$y''(2)=\frac{10}{3}\cdot\frac{1}{(2-3)^3}-\frac{4}{3}\cdot\frac{1}{2^3}=\frac{10}{3}\cdot\frac{1}{(-1)^3}-\frac{4}{3}\cdot\frac{1}{8}=-\frac{10}{3}-\frac{1}{6}=-\frac{20}{6}-\frac{1}{6}=-\frac{21}{6}=-\frac{7}{2}=-3\frac{1}{2}$

6.238.

$y=tg^2x$ w punkcie $x=0$

$y'=(tg^2x)'=2\cdot tgx\cdot(tgx)'=2\cdot tgx\cdot\frac{1}{cos^2x}=\frac{2sinx}{cos^3x}$

$y''=(\frac{2sinx}{cos^3x})'=2\cdot\frac{(sinx)'\cdot cos^3x-sinx\cdot(cos^3x)'}{(cos^3x)^2}=2\cdot\frac{cos^4x-sinx\cdot 3\cdot cos^2x\cdot cos'x}{cos^6x}=2\cdot\frac{cos^4x-3\cdot sinx\cdot cos^2x\cdot(-sinx)}{cos^6x}=$

$=2\cdot\frac{cos^4x+3\cdot sin^2x\cdot cos^2x}{cos^6x}=2\cdot\frac{cos^2x+3\cdot sin^2x}{cos^4x}=2\cdot\frac{(cos^2x+sin^2x)+2\cdot sin^2x}{cos^2x}=2\cdot\frac{1+2sin^2x}{cos^2x}$

Zatem:

$y''(0)=2\cdot\frac{1+2\cdot 0^2}{1^2}=2\cdot\frac{1}{1}=2$

6.239.

$y = ln(x+\sqrt{x^2+1})$, w punkcie $x=0$

$y' = [ln(x+\sqrt{x^2+1})]' = \frac{1}{x+\sqrt{x^2+1}} \cdot (x+\sqrt{x^2+1})' = \frac{1+[(x^2+1)^{\frac{1}{2}}]'}{x+\sqrt{x^2+1}} = \frac{1+\frac{1}{2}\cdot(x^2+1)^{-\frac{1}{2}}\cdot(x^2+1)'}{x+\sqrt{x^2+1}} =$

$= \frac{1+\frac{1}{2}\cdot 2x\cdot\frac{1}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{x+\sqrt{x^2+1}}{(x+\sqrt{x^2+1})\cdot\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}$

$y'' = (\frac{1}{\sqrt{x^2+1}})' = [(x^2+1)^{-\frac{1}{2}}]' = -\frac{1}{2}\cdot(x^2+1)^{-\frac{3}{2}}\cdot(x^2+1)' = -\frac{x}{\sqrt{(x^2+1)^3}}$

Zatem: $y''(0) = -\frac{0}{\sqrt{(0^2+1)^3}} = -\frac{0}{1} = 0$

6.240.

$y = arcsinx$, w punkcie $x=0$

Na podstawie zadania **6.236** mamy:

$y' = \frac{1}{\sqrt{1-x^2}}$ oraz $y'' = \frac{x}{\sqrt{(1-x^2)^3}}$

Natomiast trzecią pochodną musimy obliczyć:

$y''' = (\frac{x}{\sqrt{(1-x^2)^3}})' = [x\cdot(1-x^2)^{-\frac{3}{2}}] = x'\cdot(1-x^2)^{-\frac{3}{2}} + x\cdot[(1-x^2)^{-\frac{3}{2}}]' = (1-x^2)^{-\frac{3}{2}} + x\cdot(-\frac{3}{2})\cdot$

$\cdot(1-x^2)^{-\frac{5}{2}}\cdot(1-x^2)' = \frac{1}{\sqrt{(1-x^2)^3}} - \frac{3}{2}x\cdot\frac{1}{\sqrt{(1-x^2)^5}}\cdot(-2x) = \frac{1}{\sqrt{(1-x^2)^3}} + \frac{3x^2}{\sqrt{(1-x^2)^5}}$

Zatem:

$y'''(0) = \frac{1}{\sqrt{(1-0^2)^3}} + \frac{3\cdot 0^2}{\sqrt{(1-0^2)^5}} = \frac{1}{1} + \frac{0}{1} = 1$

6.241.

$y = sinx\cdot cosx$ w punkcie $x=0$

Mamy: $2sinx\cdot cosx = sin(2x) \quad\Leftrightarrow\quad sinx\cdot cosx = \frac{1}{2}\cdot sin(2x)$

Zatem: $y = \frac{1}{2}\cdot sin(2x)$

$y' = [\frac{1}{2}\cdot sin(2x)]' = \frac{1}{2}\cdot cos(2x)\cdot(2x)' = \frac{1}{2}\cdot cos(2x)\cdot 2 = cos(2x)$

$y'' = [cos(2x)]' = -sin(2x)\cdot(2x)' = -2sin(2x)$

$y''' = [-2sin(2x)]' = -2\cdot[sin(2x)]' = -2cos(2x)\cdot(2x)' = -4cos(2x)$

I ostatecznie:

$y'''(0) = -4cos(2 \cdot 0) = -4cos(0) = -4$

6.242.

$y = tgx$ w punkcie $x = 0$

$y' = (tgx)' = \frac{1}{cos^2x}$, $cosx \neq 0$

$y'' = (\frac{1}{cos^2x})' = [(cosx)^{-2}]' = -2 \cdot (cosx)^{-3} \cdot (cosx)' = -2 \cdot (cosx)^{-3} \cdot (-sinx) = 2sinxcosx \cdot (cosx)^{-4} =$

$= \frac{sin2x}{cos^4x}$

$y''' = (\frac{sin2x}{cos^4x})' = \frac{(sin2x)' \cdot cos^4x - sin2x \cdot (cos^4x)'}{[(cosx)^4]^2} = \frac{cos2x \cdot (2x)' \cdot cos^4x - sin2x \cdot 4cos^3x \cdot (cosx)'}{cos^8x} = \frac{2cos2x \cdot cos^4x - sin2x \cdot 4cos^3x \cdot (-sinx)}{cos^8x} =$

$= \frac{cos^3x \cdot (2cos2x \cdot cosx + 4sin2x \cdot sinx)}{cos^8x} = \frac{2cos2x \cdot cosx + 4sin2x \cdot sinx}{cos^5x}$

Zatem:

$y'''(0) = \frac{2 \cdot cos0 \cdot cos0 + 4 \cdot sin0 \cdot sin0}{cos^5 0} = \frac{2 \cdot 1 + 4 \cdot 0 \cdot 0}{1} = \frac{2}{1} = 2$

6.243.

$y = arctgx$ w punkcie $x = 0$

$y' = (arctgx)' = \frac{1}{1+x^2}$

$y'' = (\frac{1}{1+x^2})' = [(1+x^2)^{-1}]' = -1 \cdot (1+x^2)^{-2} \cdot (1+x^2)' = -(1+x^2)^{-2} \cdot 2x$

$y''' = [-(1+x^2)^{-2} \cdot 2x]' = -2 \cdot [x \cdot (1+x^2)^{-2}]' = -2 \cdot [x' \cdot (1+x^2)^{-2} + x \cdot ((1+x^2)^{-2})'] =$

$= -2 \cdot [(1+x^2)^{-2} + x \cdot (-2) \cdot (1+x^2)^{-3} \cdot (1+x^2)'] = -2 \cdot [(1+x^2)^{-2} - 2x \cdot (1+x^2)^{-3} \cdot 2x] =$

$= -2 \cdot [(1+x^2)^{-2} - 4x^2 \cdot (1+x^2)^{-3}]$

Zatem:

$y'''(0) = -2 \cdot [(1+0^2)^{-2} - 4 \cdot 0^2 \cdot (1+0^2)^{-3}] = -2 \cdot (1-0) = -2$

6.244.

$y = sin^2x$ w punkcie $x = 0$

$y' = [(sinx)^2]' = 2 \cdot sinx \cdot (sinx)' = 2 \cdot sinx \cdot cosx = sin2x$

$y'' = (sin2x)' = cos2x \cdot (2x)' = 2cos2x$

$y''' = (2cos2x)' = 2 \cdot (-sin2x) \cdot (2x)' = -4sin2x$

$y^4 = (-4sin2x)' = -4 \cdot cos2x \cdot (2x)' = -4cos2x \cdot 2 = -8cos2x$

Zatem:

$y^4(0) = -8 \cdot cos(2 \cdot 0) = -8cos0 = -8 \cdot 1 = -8$

6.245.

$s = 3t - t^4 \qquad\qquad\qquad , \text{dla } t = 1$

$v(t) = \frac{ds}{dt} = (3t - t^4)' = 3 - 4t^3 \qquad \Rightarrow \qquad v(1) = 3 - 4 \cdot 1^3 = 3 - 4 = -1$

$a(t) = \frac{d^2s}{dt^2} = (3 - 4t^3)' = -4 \cdot 3 \cdot t^2 = -12t^2 \qquad \Rightarrow \qquad a(1) = -12 \cdot 1^2 = -12$

6.246.

$s = t^3 + 8t^2 + 5 \qquad\qquad\qquad , \text{dla } t = -2$

$v(t) = \frac{ds}{dt} = (t^3 + 8t^2 + 5)' = 3t^2 + 8 \cdot 2t = 3t^2 + 16t$

$v(-2) = 3 \cdot (-2)^2 + 16 \cdot (-2) = 3 \cdot 4 - 32 = 12 - 32 = -20$

$a(t) = \frac{d^2s}{dt^2} = (3t^2 + 16t)' = 3 \cdot 2 \cdot t + 16 = 6t + 16$

$a(-2) = 6 \cdot (-2) + 16 = -12 + 16 = 4$

6.247.

$s = (t+1)^4 - 3(t+1)^3 \qquad\qquad\qquad , \text{dla } t = -1$

$v(t) = \frac{ds}{dt} = [(t+1)^4 - 3(t+1)^3]' = [(t+1)^4]' - 3[(t+1)^3]' = 4(t+1)^3 \cdot (t+1)' - 3 \cdot 3(t+1)^2 \cdot (t+1)' =$

$= 4(t+1)^3 \cdot 1 - 9(t+1)^2 \cdot 1 = 4(t+1)^3 - 9(t+1)^2$

$v(-1) = 4 \cdot (-1+1)^3 - 9(-1+1)^2 = 4 \cdot 0 - 9 \cdot 0 = 0$

$a(t) = \frac{d^2s}{dt^2} = [4(t+1)^3 - 9(t+1)^2]' = 4[(t+1)^3]' - 9[(t+1)^2]' = 4 \cdot 3 \cdot (t+1)^2 \cdot (t+1)' -$

$-9 \cdot 2 \cdot (t+1) \cdot (t+1)' = 12(t+1)^2 \cdot 1 - 18(t+1) \cdot 1 = 12(t+1)^2 - 18(t+1)$

$a(-1) = 12(-1+1)^2 - 18(-1+1) = 12 \cdot 0 - 18 \cdot 0 = 0$

6.248.

$s = 16t - \frac{1}{t^3}$ dla $t = -\frac{1}{2}$

$v(t) = \frac{ds}{dt} = (16t - \frac{1}{t^3})' = 16 - (t^{-3})' = 16 - (-3) \cdot t^{-4} = 16 + 3t^{-4}$

$v(-\frac{1}{2}) = 16 + 3 \cdot (-\frac{1}{2})^{-4} = 16 + 3 \cdot [(-2)^{-1}]^{-4} = 16 + 3 \cdot (-2)^4 = 16 + 3 \cdot 16 = 4 \cdot 16 = 64$

$a(t) = \frac{d^2s}{dt^2} = (16 + 3t^{-4}) = 0 + 3 \cdot (-4) \cdot t^{-5} = -12t^{-5}$

$a(-\frac{1}{2}) = -12 \cdot (-\frac{1}{2})^{-5} = -12 \cdot (-2)^5 = -12 \cdot (-32) = 384$

6.249.

$s = t^2 + t^{-1} + 3$ dla $t = \frac{1}{2}$

$v(t) = \frac{ds}{dt} = (t^2 + t^{-1} + 3)' = 2t - 1 \cdot t^{-2} + 0 = 2t - t^{-2}$

$v(\frac{1}{2}) = 2 \cdot \frac{1}{2} - (\frac{1}{2})^{-2} = 1 - 2^2 = 1 - 4 = -3$

$a(t) = \frac{d^2s}{dt^2} = (2t - t^{-2})' = 2 - (-2) \cdot t^{-3} = 2 + 2t^{-3}$

$a(\frac{1}{2}) = 2 + 2 \cdot (\frac{1}{2})^{-3} = 2 + 2 \cdot 2^3 = 2 + 2^4 = 2 + 16 = 18$

6.250.1

Końcowa jedynka w numerze powyżej oznacza pierwszy wariant tego zadania, gdyż na podstawie zapisu w książce nie jestem pewien jak wzór dany w zadaniu ma wyglądać. W następnej kolejności będzie rozwiązanie drugiego wariantu.

$s = \sqrt[3]{3t^2} - \sqrt{3}t$ dla $t = 4$

$v(t) = \frac{ds}{dt} = (\sqrt[3]{3t^2} - \sqrt{3}t)' = [(3t^2)^{\frac{1}{3}}]' - \sqrt{3} \cdot t' = \frac{1}{3} \cdot (3t^2)^{\frac{1}{3}-1} \cdot (3t^2)' - \sqrt{3} \cdot 1 =$

$= \frac{1}{3} \cdot (3t^2)^{-\frac{2}{3}} \cdot 3 \cdot 2t - \sqrt{3} = 2t \cdot (3t^2)^{-\frac{2}{3}} - \sqrt{3}$

$v(4) = 2 \cdot 4 \cdot (3 \cdot 4^2)^{-\frac{2}{3}} - \sqrt{3} = 8 \cdot (3 \cdot 16)^{-\frac{2}{3}} - \sqrt{3} = 8 \cdot 48^{-\frac{2}{3}} - \sqrt{3} = \frac{8}{\sqrt[3]{48^2}} - \sqrt{3}$

$a(t) = \frac{d^2s}{dt^2} = [2t \cdot (3t^2)^{-\frac{2}{3}} - \sqrt{3}]' = 2t' \cdot (3t^2)^{-\frac{2}{3}} + 2t \cdot [(3t^2)^{-\frac{2}{3}}]' - 0 =$

$= 2 \cdot (3t^2)^{-\frac{2}{3}} - 2t \cdot \frac{2}{3} \cdot (3t^2)^{-\frac{5}{3}} \cdot 3 \cdot 2t = 2 \cdot (3t^2)^{-\frac{2}{3}} - 8t^2 \cdot (3t^2)^{-\frac{5}{3}}$

$a(4) = 2 \cdot (3 \cdot 4^2)^{-\frac{2}{3}} - 8 \cdot 4^2 \cdot (3 \cdot 4^2)^{-\frac{5}{3}} = 2 \cdot 48^{-\frac{2}{3}} - 128 \cdot 48^{-\frac{5}{3}} = \frac{2}{\sqrt[3]{48^2}} - \frac{128}{\sqrt[3]{48^5}} = \frac{2}{\sqrt[3]{48^2}} - \frac{128}{48 \cdot \sqrt[3]{48^2}} =$

$= \frac{1}{\sqrt[3]{48^2}}(2 - \frac{8}{3}) = -\frac{2}{3\sqrt[3]{48^2}}$

6.250.2

Końcowa dwójka w numerze powyżej oznacza drugi wariant tego zadania, gdyż na podstawie zapisu w książce nie jestem pewien jak wzór dany w zadaniu ma wyglądać.

$s = \sqrt[3]{3t^2} - \sqrt{3t} \qquad\qquad$ dla $t = 4$

$v(t) = \frac{ds}{dt} = (\sqrt[3]{3t^2} - \sqrt{3t})' = [(3t^2)^{\frac{1}{3}}]' - [(3t)^{\frac{1}{2}}]' = \frac{1}{3} \cdot (3t^2)^{\frac{1}{3}-1} \cdot (3t^2)' - \frac{1}{2} \cdot (3t)^{\frac{1}{2}-1} \cdot (3t)' =$

$= \frac{3 \cdot 2t}{3} \cdot (3t^2)^{-\frac{2}{3}} - \frac{3}{2} \cdot (3t)^{-\frac{1}{2}} = 2t \cdot (3t^2)^{-\frac{2}{3}} - \frac{3}{2} \cdot (3t)^{-\frac{1}{2}}$

$v(4) = 2 \cdot 4 \cdot (3 \cdot 4^2)^{-\frac{2}{3}} - \frac{3}{2} \cdot (3 \cdot 4)^{-\frac{1}{2}} = 8 \cdot 48^{-\frac{2}{3}} - \frac{3}{2} \cdot 12^{-\frac{1}{2}} = \frac{8}{\sqrt[3]{48^2}} - \frac{3}{2 \cdot \sqrt{12}}$

$a(t) = \frac{d^2s}{dt^2} = [2t \cdot (3t^2)^{-\frac{2}{3}} - \frac{3}{2} \cdot (3t)^{-\frac{1}{2}}]' = [2t \cdot (3t^2)^{-\frac{2}{3}}]' - \frac{3}{2} \cdot [(3t)^{-\frac{1}{2}}]' =$

$= 2t' \cdot (3t^2)^{-\frac{2}{3}} + 2t \cdot (-\frac{2}{3}) \cdot (3t^2)^{-\frac{2}{3}-1} \cdot (3t^2)' - \frac{3}{2} \cdot (-\frac{1}{2}) \cdot (3t)^{-\frac{1}{2}-1} \cdot (3t)' =$

$= 2 \cdot (3t^2)^{-\frac{2}{3}} - \frac{4}{3}t \cdot (3t^2)^{-\frac{5}{3}} \cdot 3 \cdot 2t + \frac{3}{4} \cdot (3t)^{-\frac{3}{2}} \cdot 3 = 2 \cdot (3t^2)^{-\frac{2}{3}} - 8t^2 \cdot (3t^2)^{-\frac{5}{3}} + \frac{9}{4} \cdot (3t)^{-\frac{3}{2}}$

$a(4) = 2 \cdot (3 \cdot 4^2)^{-\frac{2}{3}} - 8 \cdot 4^2 \cdot (3 \cdot 4^2)^{-\frac{5}{3}} + \frac{9}{4} \cdot (3 \cdot 4)^{-\frac{3}{2}} = 2 \cdot 48^{-\frac{2}{3}} - 128 \cdot 48^{-\frac{5}{3}} + \frac{9}{4} \cdot 12^{-\frac{3}{2}} =$

$= \frac{2}{\sqrt[3]{48^2}} - \frac{128}{48 \cdot \sqrt[3]{48^2}} + \frac{9}{4} \cdot \frac{1}{\sqrt{12^3}} = \frac{2}{\sqrt[3]{48^2}} - \frac{8}{3 \cdot \sqrt[3]{48^2}} + \frac{9}{4 \cdot 12 \cdot \sqrt{12}} = \frac{2}{\sqrt[3]{48^2}} - \frac{8}{3 \cdot \sqrt[3]{48^2}} + \frac{3}{16 \cdot \sqrt{12}}$