6.115.

$y = \sqrt{sinx + \sqrt{x + 2\sqrt{x}}}$, dla $x \geqslant 0$

Pochodną obliczamy korzystając z wzorów 6.1.3, 6.1.4, 6.1.7, 6.1.10, 6.1.11:

Mamy: $y = \sqrt{u} = u^{\frac{1}{2}}$, gdzie $u = sinx + \sqrt{x + 2\sqrt{x}}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2} \cdot u^{\frac{1}{2}-1} \cdot [(sinx)' + (\sqrt{x + 2\sqrt{x}})'] = \frac{1}{2}u^{-\frac{1}{2}} \cdot (cosx + ((x + 2\sqrt{x})^{\frac{1}{2}})') = \frac{1}{2} \cdot \frac{1}{\sqrt{sinx+\sqrt{x+2\sqrt{x}}}} \cdot$
$\cdot (cosx + ((x + 2\sqrt{x})^{\frac{1}{2}})')$

Przyjmijmy teraz:

$v = (x + 2\sqrt{x})^{\frac{1}{2}} = w^{\frac{1}{2}}$, gdzie $w = x + 2\sqrt{x}$

$\frac{dv}{dx} = \frac{dv}{dw} \cdot \frac{dw}{dx} = \frac{1}{2} \cdot w^{-\frac{1}{2}} \cdot (x + 2\sqrt{x})' = \frac{1}{2w^{\frac{1}{2}}} \cdot (x' + (2x^{\frac{1}{2}})') = \frac{1}{2\sqrt{x+2\sqrt{x}}} \cdot (1 + 2 \cdot \frac{1}{2} \cdot x^{\frac{1}{2}-1}) =$
$= \frac{1+x^{-\frac{1}{2}}}{2\sqrt{x+2\sqrt{x}}} = \frac{1+\frac{1}{\sqrt{x}}}{2\sqrt{x+2\sqrt{x}}}$

A więc ostatecznie mamy:

$y' = \frac{dy}{dx} = \frac{cosx+\frac{1+\frac{1}{\sqrt{x}}}{2\sqrt{x+2\sqrt{x}}}}{2\sqrt{sinx+\sqrt{x+2\sqrt{x}}}}$, dla $x > 0$ i $sinx + \sqrt{x + 2\sqrt{x}} \neq 0$

6.116.

$y = \sqrt{1 + tg(x + \frac{1}{x})}$, dla $1 + tg(x + \frac{1}{x}) \geqslant 0$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.4, 6.1.7, 6.1.10, 6.1.13:

Mamy $y = \sqrt{u} = u^{\frac{1}{2}}$, gdzie $u = 1 + tg(x + \frac{1}{x})$

i dalej $u = 1 + tg(v)$, gdzie $v = x + \frac{1}{x}$

Zatem:

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} = \frac{1}{2}u^{\frac{1}{2}-1} \cdot (0 + \frac{1}{cos^2 v}) \cdot (1 \cdot x^{1-1} + (-1) \cdot x^{-1-1}) = \frac{1}{2}u^{-\frac{1}{2}} \cdot \frac{1}{cos^2 v} \cdot (1 - x^{-2}) =$
$= \frac{1}{2\sqrt{1+tg(x+\frac{1}{x})}} \cdot \frac{1}{cos^2(x+\frac{1}{x})} \cdot (1 - \frac{1}{x^2}) = \frac{1-\frac{1}{x^2}}{2\sqrt{1+tg(x+\frac{1}{x})} \cdot cos^2(x+\frac{1}{x})} = \frac{x^2-1}{2x^2\sqrt{1+tg(x+\frac{1}{x})} \cdot cos^2(x+\frac{1}{x})}$

6.117.

$z = \frac{3tgu-tg^3u}{1-3tg^2u}$, dla $1 - 3tg^2u \neq 0$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.3, 6.1.4, 6.1.6, 6.1.7, 6.1.10, 6.1.13, oraz

$sin^2x + cos^2x = 1$ i $cos3x = cosx(4cos^2u - 3)$

Najpierw obliczmy pochodną funkcji $y = tg^n x$, dla $n \in N$:

$y = (tgx)^n = v^n$, gdzie $v = tgx$

$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx} = n \cdot v^{n-1} \cdot \frac{1}{cos^2x} = n \cdot tg^{n-1}x \cdot \frac{1}{cos^2x}$

Zatem:

$$z' = \frac{(3tgu-tg^3u)' \cdot (1-3tg^2u)-(3tgu-tg^3u)(1-3tg^2u)'}{(1-3tg^2u)^2} = \frac{((3tgu)'-(tg^3u)') \cdot (1-3tg^2u)-(3tgu-tg^3u)(0-(3tg^2u)')}{(1-3tg^2u)^2} =$$
$$= \frac{(\frac{3}{cos^2u}-3tg^2u \cdot \frac{1}{cos^2u}) \cdot (1-3tg^2u)-(3tgu-tg^3u)(-3 \cdot 2 \cdot tgu \cdot \frac{1}{cos^2u})}{(1-3tg^2u)^2} = \frac{\frac{3}{cos^2u}-\frac{9tg^2u}{cos^2u}-\frac{3tg^2u}{cos^2u}+\frac{9tg^4u}{cos^2u}+\frac{18tg^2u}{cos^2u}-\frac{6tg^4u}{cos^2u}}{(1-3tg^2u)^2} =$$
$$= \frac{3+6tg^2u+3tg^4u}{cos^2u \cdot (1-3tg^2u)^2} = \frac{3(1+2tg^2u+tg^4u)}{cos^2u \cdot (1-3tg^2u)^2} = \frac{3(1+tg^2u)^2}{cos^2u \cdot (1-3tg^2u)^2} = \frac{3(\frac{1}{cos^2u})^2}{cos^2u \cdot (1-3tg^2u)^2} = \frac{3}{cos^4u \cdot cos^2u \cdot (1-\frac{3sin^2u}{cos^2u})^2} =$$
$$= \frac{3}{cos^2u \cdot (cos^2u \cdot (1-\frac{3sin^2u}{cos^2u}))^2} = \frac{3}{cos^2u \cdot (cos^2u-3sin^2u)^2} = \frac{3}{cos^2u \cdot (cos^2u-3(1-cos^2u))^2} = \frac{3}{cos^2u \cdot (cos^2u+3cos^2u-3)^2} =$$
$$= \frac{3}{cos^2u \cdot (4cos^2u-3)^2} = \frac{3}{(cosu(4cos^2u-3))^2} = \frac{3}{cos^2 3u}$$

6.118.

$z = tgu - ctgu - 2u$

Pochodną obliczamy korzystając z wzorów 6.1.3, 6.1.4, 6.1.10, 6.1.13, 6.1.14:

$z' = (tgu)' - (ctgu)' - 2u' = (1 + tg^2u) - (-(1 + ctg^2u)) - 2 = 1 + tg^2u + 1 + ctg^2u - 2 =$

$= tg^2u + ctg^2u + 2 - 2 = tg^2u + ctg^2u$

6.119.

$y = (4sinx - 8sin^3x)cosx$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.3, 6.1.4, 6.1.5, 6.1.7, 6.1.10, 6.1.11

oraz z wzorów:

$sin^n x = n \cdot sin^{n-1}x \cdot cosx$, wyprowadzonego w zadaniu 6.106,

$sin^2x + cos^2x = 1$,

$sin2x = 2sinxcosx$,

$cos2x = cos^2x - sin^2x$

$y = 2sinxcosx \cdot (2 - 4sin^2x) = sin2x(2 - 4sin^2x)$

$y' = (sin2x)'(2-4sin^2x) + sin2x \cdot (2-4sin^2x)' = cos2x \cdot (2x)' \cdot (2-4sin^2x) + sin2x \cdot (-4) \cdot (sin^2x)' =$

$= 2cos2x \cdot (2 - 4sin^2x) - 4sin2x \cdot 2 \cdot sinxcosx = 4cos2x(1 - 2sin^2x) - 4sin2x \cdot sin2x = 4cos2x \cdot$

$\cdot (sin^2x + cos^2x - 2sin^2x) - 4sin^2 2x = 4cos2x \cdot (cos^2x - sin^2x) - 4sin^2 2x = 4cos2x \cdot cos2x - 4sin^2 2x =$

$= 4(cos^2 2x - sin^2 2x) = 4cos4x$

6.120.

$y = arctg3x$

Pochodną obliczamy korzystając z wzorów 6.1.3, 6.1.8, 6.1.10, 6.1.13:

$y = arctg3x \quad \Leftrightarrow \quad 3x = tgy \quad \Leftrightarrow \quad x = \frac{1}{3}tgy$

Zatem: $\frac{dy}{dx} = 1 : \frac{dx}{dy} = \frac{1}{\frac{1}{3}(tgy)'} = \frac{3}{1+tg^2y} = \frac{3}{1+(tg(arctg3x))^2} = \frac{3}{1+(3x)^2} = \frac{3}{1+9x^2}$

6.121.

$y = 7arctg(\frac{1}{2}x)$

Pochodną obliczamy korzystając z wzorów 6.1.3, 6.1.8, 6.1.10, 6.1.13:

$y = 7z$, gdzie $z = arctg(\frac{1}{2}x)$

$z = arctg(\frac{1}{2}x) \quad \Leftrightarrow \quad \frac{1}{2}x = tgz \quad \Leftrightarrow \quad x = 2tgz$

Zatem: $\frac{dz}{dx} = 1 : \frac{dx}{dz} = \frac{1}{2(tgz)'} = \frac{1}{2 \cdot (1+tg^2z)} = \frac{1}{2+2 \cdot (tg(arctg(\frac{1}{2}x)))^2} = \frac{1}{2} \cdot \frac{1}{1+(\frac{1}{2}x)^2} = \frac{1}{2} \cdot \frac{1}{1+\frac{1}{4}x^2} = \frac{1}{2} \cdot \frac{4}{4+x^2} = \frac{2}{4+x^2}$

I ostatecznie: $y' = 7 \cdot (arctg(\frac{1}{2}x))' = 7 \cdot \frac{2}{4+x^2} = \frac{14}{4+x^2}$

6.122.

$x = arcsin(1 - t)$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.4, 6.1.7, 6.1.10, 6.1.15:

Mamy $\quad x = arcsin u, \quad$ gdzie $u = 1 - t$

Zatem $\quad \frac{dx}{dt} = \frac{dx}{du} \cdot \frac{du}{dt} = \frac{1}{\sqrt{1-u^2}} \cdot (0 - 1 \cdot t^{1-1}) = \frac{1}{\sqrt{1-(1-t)^2}} \cdot (0-1) = -\frac{1}{\sqrt{1-(1-2t+t^2)}} = -\frac{1}{\sqrt{t(2-t)}}$

dla $\quad t > 0 \quad \wedge \quad 2 - t > 0 \qquad \vee \qquad t < 0 \quad \wedge \quad 2 - t < 0$

$\quad t > 0 \quad \wedge \quad t < 2 \qquad \vee \qquad t < 0 \quad \wedge \quad t > 2$

$$\Updownarrow$$

$$t \in (0;2)$$

6.123.

$x = arccos\sqrt{1-t^2}$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.4, 6.1.7, 6.1.10, 6.1.16:

Mamy $x = arccos u, \quad$ gdzie $u = \sqrt{1-t^2}$

i dalej $u = \sqrt{v} = v^{\frac{1}{2}}, \quad$ gdzie $v = 1 - t^2$

Zatem:

$\frac{dx}{dt} = \frac{dx}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dt} = \frac{-1}{\sqrt{1-u^2}} \cdot \frac{1}{2} \cdot v^{\frac{1}{2}-1} \cdot (0 - 2t^{2-1}) = \frac{-1}{\sqrt{1-(\sqrt{1-t^2})^2}} \cdot \frac{1}{2 \cdot \sqrt{1-t^2}} \cdot (-2t) = \frac{1}{\sqrt{1-(1-t^2)}} \cdot \frac{1}{2 \cdot \sqrt{1-t^2}} \cdot 2t =$
$= \frac{2t}{\sqrt{t^2} \cdot 2\sqrt{1-t^2}} = \frac{t}{|t| \cdot \sqrt{1-t^2}}, \qquad$ dla $t^2 < 1 \quad \wedge \quad t \neq 0 \quad \Leftrightarrow \quad t \in (-1;1) \ \wedge \ t \neq 0$

6.124.

$x = arcsin\sqrt{t^3}, \qquad -1 < \sqrt{t^3} < 1 \quad \wedge \quad t^3 \geqslant 0$

Pochodną obliczamy korzystając z wzorów 6.1.7, 6.1.10, 6.1.15:

Mamy: $\quad x = arcsin u, \quad$ gdzie $u = \sqrt{t^3} = t^{\frac{3}{2}}$

Zatem: $\frac{dx}{dt} = \frac{dx}{du} \cdot \frac{du}{dt} = \frac{1}{\sqrt{1-u^2}} \cdot \frac{3}{2} \cdot t^{\frac{3}{2}-1} = \frac{1}{\sqrt{1-(\sqrt{t^3})^2}} \cdot \frac{3}{2} \cdot t^{\frac{1}{2}} = \frac{3\sqrt{t}}{2\sqrt{1-t^3}},$

dla $1 - t^3 > 0 \ \wedge \ t \geqslant 0 \quad \Leftrightarrow \quad t^3 < 1 \quad \wedge \ t \geqslant 0 \quad \Leftrightarrow \quad t \in [0;1)$

6.125.

$x = arcsin\frac{1}{t}, \qquad t \neq 0 \quad \wedge \quad -1 < \frac{1}{t} < 1$

Pochodną obliczamy korzystając z wzorów 6.1.7, 6.1.10, 6.1.15:

Mamy $x = arcsinu$, gdzie $u = \frac{1}{t} = t^{-1}$

Zatem $\frac{dx}{dt} = \frac{dx}{du} \cdot \frac{du}{dt} = \frac{1}{\sqrt{1-u^2}} \cdot (-1) \cdot t^{-1-1} = \frac{-1}{t^2 \cdot \sqrt{1-(\frac{1}{t})^2}} = \frac{-1}{t^2 \cdot \sqrt{\frac{t^2-1}{t^2}}} = \frac{-1}{t^2 \cdot \frac{1}{|t|} \cdot \sqrt{t^2-1}} = \frac{-|t|}{|t|^2 \cdot \sqrt{t^2-1}} =$
$= \frac{-1}{|t| \cdot \sqrt{t^2-1}}$, dla $t^2 - 1 > 0 \quad \Leftrightarrow \quad t^2 > 1 \quad \Leftrightarrow \quad |t| > 1$

6.126.

$y = arcsinx + arcsin\sqrt{1-x^2}, \qquad 0 < x < 1 \ (1)$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.4, 6.1.7, 6.1.10, 6.1.15:

Mamy $y = g(x) + h(x)$, gdzie:

$g(x) = arcsinx,$

$h(x) = arcsin\sqrt{1-x^2}$

$g'(x) = \frac{1}{\sqrt{1-x^2}}$

$h = h(x) = arcsinu, \qquad$ gdzie $u = \sqrt{1-x^2}$

i dalej $u = \sqrt{v} = v^{\frac{1}{2}}, \qquad$ gdzie $v = 1 - x^2$

Zatem: $h'(x) = \frac{dh}{dx} = \frac{dh}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} = \frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{2} \cdot v^{\frac{1}{2}-1} \cdot (0 - 2x^{2-1}) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{2 \cdot \sqrt{v}} \cdot (-2x) =$
$= \frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}} \cdot \frac{-x}{\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-(1-x^2)} \cdot \sqrt{1-x^2}} = \frac{-x}{\sqrt{x^2} \cdot \sqrt{1-x^2}} =^{(1)} \frac{-x}{x \cdot \sqrt{1-x^2}} = \frac{-1}{\sqrt{1-x^2}}$
Ostatecznie $y' = g'(x) + h'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$

6.127.

$x = arcsin(2t\sqrt{1-t^2}) \qquad 1 - t^2 \geqslant 0 \Leftrightarrow t^2 \leqslant 1 \Leftrightarrow t \in [-1; 1]$ oraz

$2t\sqrt{1-t^2} \in (-1; 1)$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.3, 6.1.4, 6.1.5, 6.1.7, 6.1.10, 6.1.15:

Mamy $x = arcsinu, \qquad$ gdzie $u = 2t\sqrt{1-t^2}$

i dalej $u = f(t) \cdot g(t)$, gdzie

$f(t) = 2t$

$g(t) = \sqrt{1-t^2} = (1-t^2)^{\frac{1}{2}}$

$g(t) = v^{\frac{1}{2}}$, gdzie $v = 1 - t^2$

$f'(t) = 2 \cdot 1 \cdot t^{1-1} = 2 \cdot 1 = 2$

$g'(t) = (v^{\frac{1}{2}})' \cdot (1-t^2)' = \frac{1}{2} \cdot v^{\frac{1}{2}-1} \cdot (0 - 2t^{2-1}) = \frac{1}{2v^{\frac{1}{2}}} \cdot (-2t) = -\frac{t}{\sqrt{v}} = -\frac{t}{\sqrt{1-t^2}}$

Zatem $x' = (arcsinu)' \cdot u' = \frac{1}{\sqrt{1-u^2}} \cdot (f'(t) \cdot g(t) + f(t) \cdot g'(t)) = \frac{1}{\sqrt{1-(2t\sqrt{1-t^2})^2}} \cdot (2 \cdot \sqrt{1-t^2} + 2t \cdot \frac{-t}{\sqrt{1-t^2}}) =$
$= \frac{1}{\sqrt{1-4t^2 \cdot (1-t^2)}} \cdot (2 \cdot \sqrt{1-t^2} - \frac{2t^2}{\sqrt{1-t^2}}) = \frac{1}{\sqrt{1-4t^2 \cdot (1-t^2)}} \cdot \frac{2 \cdot (\sqrt{1-t^2})^2 - 2t^2}{\sqrt{1-t^2}} = \frac{2 \cdot (1-t^2) - 2t^2}{\sqrt{1-t^2} \cdot \sqrt{1-4t^2 \cdot (1-t^2)}} =$
$= \frac{2-2t^2-2t^2}{\sqrt{1-t^2} \cdot \sqrt{1-4t^2 \cdot (1-t^2)}} = \frac{2 \cdot (1-2t^2)}{\sqrt{1-t^2} \cdot \sqrt{1-4t^2 \cdot (1-t^2)}} = \frac{2 \cdot (1-2t^2)}{\sqrt{1-t^2} \cdot \sqrt{1-4t^2+4t^4}} = \frac{2 \cdot (1-2t^2)}{\sqrt{1-t^2} \cdot \sqrt{(1-2t^2)^2}} =$

$$= \frac{2 \cdot (1-2t^2)}{\sqrt{1-t^2} \cdot |1-2t^2|} = \begin{cases} \frac{2}{\sqrt{1-t^2}} & dla\ 1 - 2t^2 \geqslant 0 \\ \frac{-2}{\sqrt{1-t^2}} & dla\ 1 - 2t^2 < 0 \end{cases}$$

6.128.

$y = arctg(x - \sqrt{x^2+1})$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.4, 6.1.7, 6.1.10, 6.1.17:

Mamy $y = arctgu$, gdzie $u = x - \sqrt{x^2+1}$

Zatem $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{1+u^2} \cdot (x - \sqrt{x^2+1})' = \frac{1}{1+(x-\sqrt{x^2+1})^2} \cdot (x' - (\sqrt{x^2+1})') =$
$= \frac{1}{1+(x-\sqrt{x^2+1})^2} \cdot (1 - \frac{1}{2} \cdot (x^2+1)^{\frac{1}{2}-1} \cdot (x^2+1)') = \frac{1}{1+(x-\sqrt{x^2+1})^2} \cdot (1 - \frac{1}{2 \cdot \sqrt{x^2+1}} \cdot (2x^{2-1} + 0)) =$
$= \frac{1}{1+(x-\sqrt{x^2+1})^2} \cdot (1 - \frac{2x}{2 \cdot \sqrt{x^2+1}}) = \frac{1}{1+(x-\sqrt{x^2+1})^2} \cdot \frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}-x}{(1+x^2-2x\sqrt{x^2+1}+x^2+1) \cdot \sqrt{x^2+1}} = \frac{L}{M}$
$L = \sqrt{x^2+1} - x$

$M = (1 + x^2 - 2x\sqrt{x^2+1} + x^2 + 1) \cdot \sqrt{x^2+1} = (2 + 2x^2 - 2x\sqrt{x^2+1}) \cdot \sqrt{x^2+1} =$

$= 2(x^2 + 1 - x\sqrt{x^2+1}) \cdot \sqrt{x^2+1} = 2(\sqrt{x^2+1}^2 - x\sqrt{x^2+1}) \cdot \sqrt{x^2+1} =$

$= 2\sqrt{x^2+1} \cdot (\sqrt{x^2+1} - x) \cdot \sqrt{x^2+1} = 2(x^2+1) \cdot (\sqrt{x^2+1} - x)$

A więc ostatecznie:

$y' = \frac{L}{M} = \frac{\sqrt{x^2+1}-x}{2(x^2+1) \cdot (\sqrt{x^2+1}-x)} = \frac{1}{2(x^2+1)}$

6.129.

$y = arctg(\sqrt{x^2-1}) - \frac{lnx}{\sqrt{x^2-1}}$ dla $x > 0 \ \wedge \ x^2 - 1 > 0 \ \Leftrightarrow \ x^2 > 1 \ \Leftrightarrow \ x > 1$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.4, 6.1.6, 6.1.7, 6.1.10, 6.1.17, 6.1.21:

Mamy: $y = u - v$, gdzie:

$u = arctg\sqrt{x^2-1}$

$v = \frac{lnx}{\sqrt{x^2-1}}$

$y' = u' + v'$ (1)

$u' = (arctg\sqrt{x^2-1})' = \frac{1}{1+\sqrt{x^2-1}^2} \cdot ((x^2-1)^{\frac{1}{2}})' = \frac{1}{1+x^2-1} \cdot \frac{1}{2} \cdot (x^2-1)^{\frac{1}{2}-1} \cdot (x^2-1)' = \frac{1}{x^2} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x^2-1}} \cdot 2x =$

$= \frac{1}{x \cdot \sqrt{x^2-1}}$

$v' = (\frac{lnx}{\sqrt{x^2-1}})' = \frac{(lnx)' \cdot \sqrt{x^2-1} - lnx \cdot (\sqrt{x^2-1})'}{\sqrt{x^2-1}^2} = \frac{\frac{1}{x} \cdot \sqrt{x^2-1} - lnx \cdot \frac{x}{\sqrt{x^2-1}}}{\sqrt{x^2-1}^2}$

Zatem $y' = \frac{1}{x \cdot \sqrt{x^2-1}} - \frac{\frac{1}{x} \cdot \sqrt{x^2-1} - lnx \cdot \frac{x}{\sqrt{x^2-1}}}{\sqrt{x^2-1}^2} = \frac{\sqrt{x^2-1} - (\sqrt{x^2-1} - lnx \cdot \frac{x^2}{\sqrt{x^2-1}})}{x\sqrt{x^2-1}^2} = \frac{lnx \cdot \frac{x^2}{\sqrt{x^2-1}}}{x\sqrt{x^2-1}^2} = \frac{xlnx}{\sqrt{x^2-1}^3}$

6.130.

$y = xarctgx - \frac{1}{2}ln(x^2+1)$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.3, 6.1.4, 6.1.5, 6.1.7, 6.1.10, 6.1.17, 6.1.21:

Mamy $y = u - v$, gdzie

$u = xarctgx$

$v = \frac{1}{2}ln(x^2+1)$

$u' = (xarctgx)' = x' \cdot arctgx + x \cdot (arctgx)' = arctgx + x \cdot \frac{1}{1+x^2} = arctgx + \frac{x}{1+x^2}$

$v' = \frac{1}{2} \cdot (ln(x^2+1))' = \frac{1}{2} \cdot \frac{1}{x^2+1} \cdot (x^2+1)' = \frac{1}{2(x^2+1)} \cdot (2x+0) = \frac{x}{x^2+1}$

Zatem: $y' = u' - v' = (arctgx + \frac{x}{1+x^2}) - \frac{x}{x^2+1} = arctgx$