6.131.

$y=\frac{1}{5}x^5\cdot arctgx-\frac{1}{20}x^4+\frac{1}{10}x^2-\frac{1}{10}\cdot ln(1+x^2)$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.3, 6.1.4, 6.1.5, 6.1.7, 6.1.10, 6.1.17, 6.1.21

Mamy: $(\frac{1}{5}x^5\cdot arctgx)'=(\frac{1}{5}x^5)'\cdot arctgx+\frac{1}{5}x^5\cdot(arctgx)'=\frac{1}{5}\cdot 5\cdot x^4\cdot arctgx+\frac{1}{5}x^5\cdot\frac{1}{1+x^2}=$

$=x^4\cdot arctgx+\frac{x^5}{5(1+x^2)}\qquad(1)$

$(\frac{1}{20}x^4)'=\frac{1}{20}\cdot 4\cdot x^3=\frac{1}{5}x^3\qquad(2)$

$(\frac{1}{10}x^2)'=\frac{1}{10}\cdot 2\cdot x=\frac{1}{5}x\qquad(3)$

$[\frac{1}{10}\cdot ln(1+x^2)]'=\frac{1}{10}\cdot\frac{1}{1+x^2}\cdot(1+x^2)'=\frac{1}{10\cdot(1+x^2)}\cdot(0+2x)=\frac{2x}{10\cdot(1+x^2)}=\frac{x}{5\cdot(1+x^2)}\qquad(4)$

Zatem: $y'=(1)-(2)+(3)-(4)=x^4\cdot arctgx+\frac{x^5}{5(1+x^2)}-\frac{x^3}{5}+\frac{x}{5}-\frac{x}{5\cdot(1+x^2)}=$

$=x^4\cdot arctgx+\frac{x^5}{5\cdot(1+x^2)}-\frac{x^3(1+x^2)}{5\cdot(1+x^2)}+\frac{x(1+x^2)}{5\cdot(1+x^2)}-\frac{x}{5\cdot(1+x^2)}=x^4\cdot arctgx+\frac{x^5-x^3-x^5+x+x^3-x}{5\cdot(1+x^2)}=$

$=x^4\cdot arctgx+\frac{0}{5\cdot(1+x^2)}=x^4\cdot arctgx$

6.132.

$y=arcsin\frac{x}{\sqrt{1+x^2}}\qquad,\ \frac{x}{\sqrt{1+x^2}}\in(-1;1)$

Pochodną obliczamy korzystając z wzorów 6.1.2, 6.1.4, 6.1.6, 6.1.7, 6.1.10, 6.1.15

$y'=\frac{1}{\sqrt{1-(\frac{x}{\sqrt{1+x^2}})^2}}\cdot(\frac{x}{\sqrt{1+x^2}})'=\frac{1}{\sqrt{1-\frac{x^2}{1+x^2}}}\cdot\frac{x'\cdot\sqrt{1+x^2}-x\cdot(\sqrt{1+x^2})'}{(\sqrt{1+x^2})^2}=\frac{1}{\sqrt{\frac{1+x^2}{1+x^2}-\frac{x^2}{1+x^2}}}\cdot\frac{1\cdot\sqrt{1+x^2}-x\cdot[(1+x^2)^{\frac{1}{2}}]'}{1+x^2}=$

$=\frac{1}{\sqrt{\frac{1}{1+x^2}}}\cdot\frac{\sqrt{1+x^2}-x\cdot\frac{1}{2}\cdot(1+x^2)^{\frac{1}{2}-1}\cdot(1+x^2)'}{1+x^2}=\sqrt{1+x^2}\cdot\frac{\sqrt{1+x^2}-\frac{1}{2}x\cdot\frac{1}{\sqrt{1+x^2}}\cdot(0+2x)}{1+x^2}=\frac{1+x^2-\frac{1}{2}x\cdot 2x}{1+x^2}=\frac{1+x^2-x^2}{1+x^2}=\frac{1}{1+x^2}$

6.133.

$y=arccos\sqrt{\frac{1-x^2}{1+x^2}},\qquad$ dla $1-x^2\geqslant 0\quad\Leftrightarrow\quad x\in(-1;1)$

Pochodną obliczamy korzystając z wzorów: 6.1.2, 6.1.4, 6.1.6, 6.1.7, 6.1.10, 6.1.16:

Mamy:

$(\sqrt{\frac{1-x^2}{1+x^2}})'=[(\frac{1-x^2}{1+x^2})^{\frac{1}{2}}]'=\frac{1}{2}\cdot(\frac{1-x^2}{1+x^2})^{\frac{1}{2}-1}\cdot(\frac{1-x^2}{1+x^2})'=\frac{1}{2}\cdot(\frac{1-x^2}{1+x^2})^{-\frac{1}{2}}\cdot\frac{(1-x^2)'(1+x^2)-(1-x^2)(1+x^2)'}{(1+x^2)^2}=$

$\frac{1}{2}\cdot\sqrt{\frac{1+x^2}{1-x^2}}\cdot\frac{(0-2x)(1+x^2)-(1-x^2)(0+2x)}{(1+x^2)^2}=\frac{1}{2}\cdot\sqrt{\frac{1+x^2}{1-x^2}}\cdot\frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2}=\sqrt{\frac{1+x^2}{1-x^2}}\cdot\frac{-x(1+x^2)-x(1-x^2)}{(1+x^2)^2}=$

$$= \frac{(1+x^2)^{\frac{1}{2}}}{\sqrt{1-x^2}} \cdot \frac{-x-x^3-x+x^3}{(1+x^2)^2} = \frac{-2x}{\sqrt{1-x^2}\cdot(1+x^2)^{\frac{3}{2}}}$$

Zatem:

$$y' = \frac{-1}{\sqrt{1-(\sqrt{\frac{1-x^2}{1+x^2}})^2}} \cdot (\sqrt{\frac{1-x^2}{1+x^2}})' = \frac{-1}{\sqrt{1-\frac{1-x^2}{1+x^2}}} \cdot \frac{-2x}{\sqrt{1-x^2}\cdot(1+x^2)^{\frac{3}{2}}} = \frac{2x}{\sqrt{1-\frac{1-x^2}{1+x^2}}\cdot\sqrt{1-x^2}\cdot\sqrt{(1+x^2)^3}} =$$

$$= \frac{2x}{\sqrt{\frac{1+x^2}{1+x^2}-\frac{1-x^2}{1+x^2}}\cdot\sqrt{(1-x^2)(1+x^2)^3}} = \frac{2x}{\sqrt{\frac{2x^2}{1+x^2}\cdot(1+x^2)^3\cdot(1-x^2)}} = \frac{2x}{\sqrt{2x^2\cdot(1+x^2)^2\cdot(1-x^2)}} =$$

$$= \frac{\sqrt{2}\cdot\sqrt{2}\cdot x}{\sqrt{2}\cdot|x|\cdot(1+x^2)\cdot\sqrt{1-x^2}} = \begin{cases} \frac{\sqrt{2}}{(1+x^2)\cdot\sqrt{1-x^2}} & dla\, x \in (0;1) \\ \frac{-\sqrt{2}}{(1+x^2)\cdot\sqrt{1-x^2}} & dla\, x \in (-1;0) \end{cases}$$

6.134.

$y = arctg\sqrt{\frac{1-x}{1+x}}$, dla $x \neq -1 \ \wedge \ \frac{1-x}{1+x} \geqslant 0$

Pochodną obliczamy korzystając z wzorów: 6.1.2, 6.1.4, 6.1.6, 6.1.7, 6.1.10, 6.1.17

Mamy:

$y' = \frac{1}{1+(\sqrt{\frac{1-x}{1+x}})^2} \cdot (\sqrt{\frac{1-x}{1+x}})' = \frac{1}{1+\frac{1-x}{1+x}} \cdot g'(x)$, gdzie $g(x) = \sqrt{\frac{1-x}{1+x}} = (\frac{1-x}{1+x})^{\frac{1}{2}}$

$$g'(x) = \frac{1}{2} \cdot (\frac{1-x}{1+x})^{\frac{1}{2}-1} \cdot (\frac{1-x}{1+x})' = \frac{1}{2} \cdot (\frac{1-x}{1+x})^{-\frac{1}{2}} \cdot \frac{(1-x)'\cdot(1+x)-(1-x)\cdot(1+x)'}{(1+x)^2} = \frac{1}{2} \cdot (\frac{1+x}{1-x})^{\frac{1}{2}} \cdot \frac{(0-1)\cdot(1+x)-(1-x)\cdot(0+1)}{(1+x)^2} =$$

$$= \frac{1}{2} \cdot (\frac{1+x}{1-x})^{\frac{1}{2}} \cdot \frac{-1\cdot(1+x)-(1-x)\cdot 1}{(1+x)^2} = \frac{1}{2} \cdot (\frac{1+x}{1-x})^{\frac{1}{2}} \cdot \frac{-1-x-1+x}{(1+x)^2} = \frac{1}{2} \cdot (\frac{1+x}{1-x})^{\frac{1}{2}} \cdot \frac{-2}{(1+x)^2} = -(\frac{1+x}{1-x})^{\frac{1}{2}} \cdot \frac{1}{(1+x)^2}$$

Zatem:

$$y' = \frac{1}{1+\frac{1-x}{1+x}} \cdot (-1) \cdot \frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}\cdot(1+x)^2} = \frac{1}{\frac{1+x+1-x}{1+x}} \cdot (-1) \cdot \frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}\cdot(1+x)^2} = -\frac{1+x}{2} \cdot \frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}\cdot(1+x)^2} =$$

$$= \frac{-1}{2(1+x)} \cdot \frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}} = \frac{-1}{2(1+x)} \cdot \sqrt{\frac{1+x}{1-x}} = -\frac{1}{2} \cdot \sqrt{\frac{1+x}{(1+x)^2\cdot(1-x)}} = -\frac{1}{2} \cdot \sqrt{\frac{1}{(1+x)\cdot(1-x)}} = -\frac{1}{2\sqrt{(1+x)\cdot(1-x)}}$$

6.135.

$y = arctg\frac{1+x}{1-x}$, dla $x \neq 1$

Pochodną obliczamy korzystając z wzorów: 6.1.2, 6.1.4, 6.1.6, 6.1.7, 6.1.10, 6.1.17:

$$y' = \frac{1}{1+(\frac{1+x}{1-x})^2} \cdot (\frac{1+x}{1-x})' = \frac{1}{\frac{(1-x)^2+(1+x)^2}{(1-x)^2}} \cdot \frac{(1+x)'\cdot(1-x)-(1+x)\cdot(1-x)'}{(1-x)^2} = \frac{(1-x)^2}{(1-x)^2+(1+x)^2} \cdot \frac{1\cdot(1-x)-(1+x)\cdot(-1)}{(1-x)^2} =$$

$$= \frac{1-x+1+x}{(1-x)^2+(1+x)^2} = \frac{2}{1-2x+x^2+1+2x+x^2} = \frac{2}{2+2x^2} = \frac{1}{1+x^2}$$

6.136.

$y = arctg\frac{x}{1+\sqrt{1+x^2}}$

Pochodną obliczamy korzystając z wzorów: 6.1.2, 6.1.4, 6.1.6, 6.1.7, 6.1.10, 6.1.17:

Mamy: $y = arctgu,$ gdzie $u = \frac{x}{1+\sqrt{1+x^2}}$

Zatem: $y' = \frac{1}{1+u^2} \cdot u'$ (1)

$$u' = \frac{x'\cdot(1+\sqrt{1+x^2})-x\cdot(1+\sqrt{1+x^2})'}{(1+\sqrt{1+x^2})^2} = \frac{1+\sqrt{1+x^2}-x\cdot(0+\frac{1}{2}\cdot(1+x^2)^{-\frac{1}{2}}\cdot(1+x^2)'}{(1+\sqrt{1+x^2})^2} = \frac{1+\sqrt{1+x^2}-x\cdot(\frac{1}{2}\cdot\frac{1}{\sqrt{1+x^2}}\cdot 2x}{(1+\sqrt{1+x^2})^2}$$

$$(1) = \frac{1}{1+\frac{x^2}{(1+\sqrt{1+x^2})^2}} \cdot \frac{1+\sqrt{1+x^2}-\frac{x^2}{\sqrt{1+x^2}}}{(1+\sqrt{1+x^2})^2} = \frac{1}{\frac{x^2+(1+\sqrt{1+x^2})^2}{(1+\sqrt{1+x^2})^2}} \cdot \frac{1+\sqrt{1+x^2}-\frac{x^2}{\sqrt{1+x^2}}}{(1+\sqrt{1+x^2})^2} = \frac{(1+\sqrt{1+x^2})^2}{x^2+(1+\sqrt{1+x^2})^2} \cdot \frac{1+\sqrt{1+x^2}-\frac{x^2}{\sqrt{1+x^2}}}{(1+\sqrt{1+x^2})^2} =$$

$$= \frac{1+\sqrt{1+x^2}-\frac{x^2}{\sqrt{1+x^2}}}{x^2+(1+\sqrt{1+x^2})^2} = \frac{1+\sqrt{1+x^2}-\frac{x^2}{\sqrt{1+x^2}}}{x^2+1+2\sqrt{1+x^2}+(1+x^2)} = \frac{\frac{\sqrt{1+x^2}}{\sqrt{1+x^2}}+\frac{1+x^2}{\sqrt{1+x^2}}-\frac{x^2}{\sqrt{1+x^2}}}{2\cdot(x^2+\sqrt{1+x^2}+1)} = \frac{\sqrt{1+x^2}+1}{2\cdot\sqrt{1+x^2}\cdot(x^2+\sqrt{1+x^2}+1)} =$$

$$= \frac{\sqrt{1+x^2}\cdot(\sqrt{1+x^2}+1)}{2\cdot\sqrt{1+x^2}\cdot\sqrt{1+x^2}\cdot(x^2+\sqrt{1+x^2}+1)} = \frac{1+x^2+\sqrt{1+x^2}}{2\cdot(1+x^2)\cdot(x^2+\sqrt{1+x^2}+1)} = \frac{1+x^2+\sqrt{1+x^2}}{2\cdot(1+x^2)\cdot(1+x^2+\sqrt{1+x^2})} = \frac{1}{2\cdot(1+x^2)}$$

6.137.

$y = arctg\frac{\sqrt{1+x^2}-1}{x}$

Pochodną obliczamy korzystając z wzorów: 6.1.2, 6.1.4, 6.1.6, 6.1.7, 6.1.10, 6.1.17:

$y' = (arctgu)'$, gdzie $u = \frac{\sqrt{1+x^2}-1}{x}$

$y' = \frac{1}{1+u^2} \cdot u'$ (1)

$$u' = \frac{(\sqrt{1+x^2}-1)'\cdot x-(\sqrt{1+x^2}-1)\cdot x'}{x^2} = \frac{(\frac{1}{2}\cdot(1+x^2)^{\frac{1}{2}-1}\cdot(1+x^2)'-0)\cdot x-(\sqrt{1+x^2}-1)\cdot 1}{x^2} = \frac{(\frac{1}{2}\cdot(1+x^2)^{-\frac{1}{2}}\cdot(0+2x))\cdot x-\sqrt{1+x^2}+1}{x^2} =$$

$$= \frac{\frac{2x}{2\cdot\sqrt{1+x^2}}\cdot x-\frac{1+x^2}{\sqrt{1+x^2}}+\frac{\sqrt{1+x^2}}{\sqrt{1+x^2}}}{x^2} = \frac{\frac{x^2-1-x^2+\sqrt{1+x^2}}{\sqrt{1+x^2}}}{x^2} = \frac{-1+\sqrt{1+x^2}}{x^2\cdot\sqrt{1+x^2}}$$

$$(1) = \frac{1}{1+\frac{(\sqrt{1+x^2}-1)^2}{x^2}} \cdot \frac{\sqrt{1+x^2}-1}{x^2\cdot\sqrt{1+x^2}} = \frac{x^2}{x^2+(\sqrt{1+x^2}-1)^2} \cdot \frac{\sqrt{1+x^2}-1}{x^2\cdot\sqrt{1+x^2}} = \frac{1}{x^2+(1+x^2-2\sqrt{1+x^2}+1)} \cdot \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}} =$$

$$= \frac{1}{2x^2+2-2\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}} = \frac{1}{2(x^2+1-\sqrt{1+x^2})} \cdot \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}} = \frac{1}{2(1+x^2-\sqrt{1+x^2})} \cdot \frac{1+x^2-\sqrt{1+x^2}}{1+x^2} =$$

$$= \frac{1}{2(1+x^2)}$$

6.138.

$y = \frac{arctg2x}{arcctg2x}$

Pochodną obliczamy korzystając z wzorów: 6.1.3, 6.1.6, 6.1.7, 6.1.10, 6.1.17, 6.1.18

oraz z wzoru $arctgx + arcctgx = \frac{\pi}{2}$:

$y' = \frac{(arctg2x)' \cdot arcctg2x - arctg2x \cdot (arcctg2x)'}{(arcctg2x)^2} = \frac{\frac{1}{1+4x^2} \cdot (2x)' \cdot arcctg2x - arctg2x \cdot \frac{-1}{1+4x^2} \cdot (2x)'}{(arcctg2x)^2} = \frac{2 \cdot arcctg2x + 2 \cdot arctg2x}{(1+4x^2) \cdot (arcctg2x)^2} =$

$= \frac{2 \cdot (arctg2x + arcctg2x)}{(1+4x^2) \cdot (arcctg2x)^2} = \frac{2 \cdot \frac{\pi}{2}}{(1+4x^2) \cdot (arcctg2x)^2} = \frac{\pi}{(1+4x^2) \cdot (arcctg2x)^2}$

6.139.

$z = \sqrt{\frac{1-arcsiny}{1+arcsiny}}$

Pochodną obliczamy korzystając z wzorów: 6.1.2, 6.1.4, 6.1.6, 6.1.7, 6.1.10, 6.1.15

Mamy: $z = \sqrt{u}$, gdzie $u = \frac{1-arcsiny}{1+arcsiny}$

Zatem: $z' = \frac{1}{2} u^{\frac{1}{2}-1} \cdot u' = \frac{1}{2} u^{-\frac{1}{2}} \cdot u' \quad (1)$

$u' = \frac{(1-arcsiny)' \cdot (1+arcsiny) - (1-arcsiny) \cdot (1+arcsiny)'}{(1+arcsiny)^2} = \frac{(0 - \frac{1}{\sqrt{1-y^2}}) \cdot (1+arcsiny) - (1-arcsiny) \cdot (0 + \frac{1}{\sqrt{1-y^2}})}{(1+arcsiny)^2} =$

$= \frac{-\frac{1}{\sqrt{1-y^2}} - \frac{1}{\sqrt{1-y^2}} \cdot arcsiny - \frac{1}{\sqrt{1-y^2}} + \frac{1}{\sqrt{1-y^2}} \cdot arcsiny}{(1+arcsiny)^2} = \frac{-2 \cdot \frac{1}{\sqrt{1-y^2}}}{(1+arcsiny)^2} = \frac{-2}{\sqrt{1-y^2} \cdot (1+arcsiny)^2}$

$(1) = \frac{1}{2} \cdot \sqrt{\frac{1+arcsiny}{1-arcsiny}} \cdot \frac{-2}{\sqrt{1-y^2} \cdot (1+arcsiny)^2} = -\sqrt{\frac{1+arcsiny}{1-arcsiny} \cdot \frac{1}{(1+arcsiny)^2}} \cdot \frac{1}{\sqrt{1-y^2} \cdot (1+arcsiny)} =$

$= -\sqrt{\frac{1}{(1-arcsiny)(1+arcsiny)}} \cdot \frac{1}{\sqrt{1-y^2} \cdot (1+arcsiny)} = \frac{1}{\sqrt{1-y^2} \cdot \sqrt{1-arcsin^2 y} \cdot (1+arcsiny)}$

6.140.

$y = x^3 \cdot arctgx^3$

Pochodną obliczamy korzystając z wzorów: 6.1.5, 6.1.7, 6.1.10, 6.1.17

$y' = (x^3)' \cdot arctgx^3 + x^3 \cdot (arctgx^3)' = 3x^{3-1} \cdot arctgx^3 + x^3 \cdot \frac{1}{1+(x^3)^2} \cdot (x^3)' =$

$= 3x^2 \cdot arctgx^3 + x^3 \cdot \frac{1}{1+x^6} \cdot 3x^2 = 3x^2 \cdot arctgx^3 + \frac{3x^5}{1+x^6}$

6.141.

$z = \frac{arcsin4y}{1-4y} \qquad$ dla $y \neq \frac{1}{4} \quad \wedge \quad 4y \in [-\frac{\pi}{2}; \frac{\pi}{2}]$

Pochodną obliczamy korzystając z wzorów: 6.1.2, 6.1.3, 6.1.4, 6.1.6, 6.1.7, 6.1.10, 6.1.15

Obliczmy najpierw pochodne funkcji z licznika i mianownika:

$(arcsin4y)' = \frac{1}{\sqrt{1-(4y)^2}} \cdot (4y)' = \frac{1}{\sqrt{1-16y^2}} \cdot 4 = \frac{4}{\sqrt{1-16y^2}}$

$(1-4y)' = 0 - 4 = -4$

Zatem:

$z' = \frac{(arcsin4y)' \cdot (1-4y) - arcsin4y \cdot (1-4y)'}{(1-4y)^2} = \frac{\frac{4}{\sqrt{1-16y^2}} \cdot (1-4y) - arcsin4y \cdot (-4)}{(1-4y)^2} = \frac{\frac{4 \cdot (1-4y)}{\sqrt{(1-4y)(1+4y)}} + 4arcsin4y}{(1-4y)^2} =$

$= \frac{\frac{4}{\sqrt{(1-4y)(1+4y) \cdot \frac{1}{(1-4y)^2}}} + 4arcsin4y}{(1-4y)^2} = \frac{\frac{4}{\sqrt{\frac{1+4y}{1-4y}}} + 4arcsin4y}{(1-4y)^2} = \frac{4}{(1-4y)^2} \cdot (\sqrt{\frac{1-4y}{1+4y}} + arcsin4y)$

6.142.

$y = \frac{4}{\sqrt{3}} arctg[\frac{1}{\sqrt{3}}(2tg\frac{x}{2} + 1)] - x$

Pochodną obliczamy korzystając z wzorów: 6.1.2, 6.1.3, 6.1.4, 6.1.7, 6.1.10, 6.1.13,

6.1.17 oraz $2sinxcosx = sin(x+y) + sin(x-y)$, $sin^2x + cos^2x = 1$

$y' = \frac{4}{\sqrt{3}} \cdot \frac{1}{1+[\frac{1}{\sqrt{3}}(2tg\frac{x}{2}+1)]^2} \cdot [\frac{1}{\sqrt{3}}(2tg\frac{x}{2} + 1)]' - 1 = \frac{4}{\sqrt{3}} \cdot \frac{1}{1+(\frac{1}{\sqrt{3}})^2 \cdot (2tg\frac{x}{2}+1)^2} \cdot \frac{1}{\sqrt{3}} \cdot [2 \cdot (tg\frac{x}{2})' + 0] - 1 =$

$= \frac{4}{\sqrt{3}} \cdot \frac{1}{1+\frac{1}{3} \cdot (2tg\frac{x}{2}+1)^2} \cdot \frac{1}{\sqrt{3}} \cdot 2 \cdot \frac{1}{cos^2\frac{x}{2}} \cdot (\frac{1}{2}x)' - 1 = \frac{4}{\sqrt{3}} \cdot \frac{1}{1+\frac{1}{3} \cdot (2\frac{sin\frac{x}{2}}{cos\frac{x}{2}} + \frac{cos\frac{x}{2}}{cos\frac{x}{2}})^2} \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{cos^2\frac{x}{2}} - 1 =$

$= \frac{4}{\sqrt{3}} \cdot \frac{1}{1+\frac{1}{3} \cdot (\frac{2 \cdot sin\frac{x}{2} + cos\frac{x}{2}}{cos\frac{x}{2}})^2} \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{cos^2\frac{x}{2}} - 1 = \frac{4}{\sqrt{3}} \cdot \frac{1}{1+\frac{4 \cdot sin^2\frac{x}{2} + 4sin\frac{x}{2}cos\frac{x}{2} + cos^2\frac{x}{2}}{3 \cdot cos^2\frac{x}{2}}} \cdot \frac{1}{\sqrt{3} \cdot cos^2\frac{x}{2}} - 1 =$

$= \frac{4}{\sqrt{3}} \cdot \frac{1}{\frac{3 \cdot cos^2\frac{x}{2}}{3 \cdot cos^2\frac{x}{2}} + \frac{4 \cdot sin^2\frac{x}{2} + 4sin\frac{x}{2}cos\frac{x}{2} + cos^2\frac{x}{2}}{3 \cdot cos^2\frac{x}{2}}} \cdot \frac{1}{\sqrt{3} \cdot cos^2\frac{x}{2}} - 1 = \frac{4}{\sqrt{3}} \cdot \frac{3 \cdot cos^2\frac{x}{2}}{3 \cdot cos^2\frac{x}{2} + 4 \cdot sin^2\frac{x}{2} + 2 \cdot 2sin\frac{x}{2}cos\frac{x}{2} + cos^2\frac{x}{2}} \cdot \frac{1}{\sqrt{3} \cdot cos^2\frac{x}{2}} - 1 =$

$= \frac{4}{3} \cdot \frac{3}{4 \cdot cos^2\frac{x}{2} + 4 \cdot sin^2\frac{x}{2} + 2 \cdot [sin(\frac{x}{2} + \frac{x}{2}) + sin(\frac{x}{2} - \frac{x}{2})]} - 1 = \frac{4}{4 \cdot (sin^2\frac{x}{2} + cos^2\frac{x}{2}) + 2 \cdot (sinx + sin0)} - 1 = \frac{2}{2 \cdot 1 + sinx + 0} - 1 =$

$= \frac{2}{2+sinx} - 1 = \frac{2}{2+sinx} - \frac{2+sinx}{2+sinx} = \frac{2-2-sinx}{2+sinx} = -\frac{sinx}{2+sinx}$

6.143.

$y = \frac{1}{\sqrt{a^2-b^2}} \cdot arcsin\frac{acosx+b}{a+bcosx}$

Pochodną obliczamy korzystając z wzorów: 6.1.2, 6.1.3, 6.1.4, 6.1.6, 6.1.7, 6.1.11,

6.1.12, 6.1.15 oraz $sin^2x + cos^2x = 1$

Obliczmy najpierw pochodną:

$(arcsin\frac{acosx+b}{a+bcosx})' = \frac{1}{\sqrt{1-(\frac{acosx+b}{a+bcosx})^2}} \cdot (\frac{acosx+b}{a+bcosx})' = \frac{1}{\sqrt{1-\frac{a^2cos^2x+2abcosx+b^2}{a^2+2abcosx+b^2cos^2x}}} \cdot \frac{(acosx+b)' \cdot (a+bcosx)-(acosx+b) \cdot (a+bcosx)'}{(a+bcosx)^2} =$

$= \frac{1}{\sqrt{\frac{a^2+2abcosx+b^2cos^2x-(a^2cos^2x+2abcosx+b^2)}{a^2+2abcosx+b^2cos^2x}}} \cdot \frac{-asinx \cdot (a+bcosx)-(acosx+b) \cdot (-bsinx)}{a^2+2abcosx+b^2cos^2x} = \frac{\sqrt{a^2+2abcosx+b^2cos^2x}}{\sqrt{a^2-b^2+b^2cos^2x-a^2cos^2x}} \cdot$

$\cdot \frac{-a^2sinx-absinxcosx+absinxcosx+b^2sinx}{a^2+2abcosx+b^2cos^2x} = \frac{\sqrt{a^2+2abcosx+b^2cos^2x}}{\sqrt{(a^2-b^2)-cos^2x(a^2-b^2)}} \cdot \frac{-a^2sinx+b^2sinx}{\sqrt{a^2+2abcosx+b^2cos^2x} \cdot \sqrt{a^2+2abcosx+b^2cos^2x}} =$

$= \frac{1}{\sqrt{(a^2-b^2)(1-cos^2x)}} \cdot \frac{-sinx \cdot (a^2-b^2)}{\sqrt{a^2+2abcosx+b^2cos^2x}} = \frac{1}{\sqrt{a^2-b^2} \cdot \sqrt{1-cos^2x}} \cdot \frac{-sinx \cdot \sqrt{a^2-b^2} \cdot \sqrt{a^2-b^2}}{\sqrt{(a+bcosx)^2}} =$

$= \frac{-sinx \cdot \sqrt{a^2-b^2}}{\sqrt{1-cos^2x} \cdot \sqrt{(a+bcosx)^2}} = \frac{-sinx \cdot \sqrt{a^2-b^2}}{\sqrt{sin^2x} \cdot (a+bcosx)} = \frac{-sinx \cdot \sqrt{a^2-b^2}}{sinx \cdot (a+bcosx)} = \frac{-\sqrt{a^2-b^2}}{a+bcosx}$

Zatem ostatecznie:

$y' = \frac{1}{\sqrt{a^2-b^2}} \cdot \frac{-\sqrt{a^2-b^2}}{a+bcosx} = -\frac{1}{a+bcosx}$

6.144.

$y = e^{3x}$

Pochodną obliczamy korzystając z wzorów: 6.1.3, 6.1.7, 6.1.10, 6.1.19

$y' = e^{3x} \cdot (3x)' = 3e^{3x}$

6.145.

$y = 5e^{\frac{1}{2}x}$

Pochodną obliczamy korzystając z wzorów: 6.1.3, 6.1.7, 6.1.10, 6.1.19

$y' = 5 \cdot e^{\frac{1}{2}x} \cdot (\frac{1}{2}x)' = \frac{5}{2} \cdot e^{\frac{1}{2}x}$

6.146.

$y = e^x \cdot f(x)$

Pochodną obliczamy korzystając z wzorów: 6.1.5, 6.1.19

$y' = (e^x)' \cdot f(x) + e^x \cdot f'(x) = e^x \cdot f(x) + e^x \cdot f'(x) = e^x \cdot (f(x) + f'(x))$

6.147.

$y = 3e^{-2x} \cdot g(x)$

Pochodną obliczamy korzystając z wzorów: 6.1.3, 6.1.5, 6.1.10, 6.1.19

$y' = 3 \cdot [(e^{-2x})' \cdot g(x) + e^{-2x} \cdot g'(x)] = 3 \cdot [e^{-2x} \cdot (-2x)' \cdot g(x) + e^{-2x} \cdot g'(x)] = 3 \cdot [-2e^{-2x} \cdot g(x) + e^{-2x} \cdot g'(x)] =$

$= 3e^{-2x} \cdot [-2g(x) + g'(x)]$