7.11.

$x = 4t,\ y = 8(1-t)$

$\frac{dx}{dt} = (4t)' = 4$

$\frac{dy}{dt} = [8(1-t)]' = (8-8t)' = -8$

Zatem: $\frac{dy}{dx} = \frac{-8}{4} = -2$

7.12.

$x = acost,\ y = bsint$

$\frac{dx}{dt} = (acost)' = a \cdot (-sint) = -asint$

$\frac{dy}{dt} = (bsint)' = bcost$

Zatem: $\frac{dy}{dx} = \frac{bcost}{-asint} = -\frac{b}{a} \cdot ctgt$

7.13.

$x = \frac{t^2}{t-1},\ y = \frac{t}{t^2-1},$ dla $t \neq 1 \wedge t \neq -1$

$\frac{dx}{dt} = (\frac{t^2}{t-1})' = \frac{(t^2)' \cdot (t-1) - t^2 \cdot (t-1)'}{(t-1)^2} = \frac{2t \cdot (t-1) - t^2 \cdot 1}{(t-1)^2} = \frac{2t^2-2t-t^2}{(t-1)^2} = \frac{t^2-2t}{(t-1)^2} = \frac{t(t-2)}{(t-1)^2}$

$\frac{dy}{dt} = (\frac{t}{t^2-1})' = \frac{t' \cdot (t^2-1) - t \cdot (t^2-1)'}{(t^2-1)^2} = \frac{(t^2-1) - t \cdot (2t)}{(t^2-1)^2} = \frac{t^2-1-2t^2}{(t^2-1)^2} = -\frac{t^2+1}{(t^2-1)^2}$

Zatem: $\frac{dy}{dx} = -\frac{t^2+1}{(t^2-1)^2} \cdot \frac{(t-1)^2}{t(t-2)} = -\frac{(t^2+1)(t-1)^2}{[(t-1)(t+1)]^2 \cdot t(t-2)} = -\frac{(t^2+1)(t-1)^2}{(t-1)^2 \cdot (t+1)^2 \cdot t(t-2)} = \frac{-(t^2+1)}{t(t-2) \cdot (t+1)^2}$, dla $t \neq 0 \wedge t \neq 2$

7.14.

$x = \frac{1}{t+1}, \quad y = (\frac{t}{t+1})^2$ $\quad dla\, t \neq -1$

$\frac{dx}{dt} = (\frac{1}{t+1})' = \frac{1' \cdot (t+1) - 1 \cdot (t+1)'}{(t+1)^2} = \frac{0-1}{(t+1)^2} = \frac{-1}{(t+1)^2}$

$\frac{dy}{dt} = [(\frac{t}{t+1})^2]' = [\frac{t^2}{(t+1)^2}]' = \frac{(t^2)' \cdot (t+1)^2 - t^2 \cdot [(t+1)^2]'}{[(t+1)^2]^2} = \frac{2t \cdot (t+1)^2 - t^2 \cdot 2 \cdot (t+1) \cdot (t+1)'}{(t+1)^4} = \frac{2t \cdot (t+1)^2 - 2t^2 \cdot (t+1) \cdot 1}{(t+1)^4} =$

$= \frac{2t \cdot (t+1) \cdot [(t+1) - t]}{(t+1)^4} = \frac{2t \cdot (t+1) \cdot 1}{(t+1)^4} = \frac{2t}{(t+1)^3}$

Zatem: $\frac{dy}{dx} = \frac{2t}{(t+1)^3} \cdot [\frac{(t+1)^2}{-1}] = \frac{2t}{(t+1)^3} \cdot [-(t+1)^2] = -\frac{2t}{t+1}$

7.15.

$x = \frac{b}{b-t}, \qquad y = \frac{a}{a-t} \qquad\qquad dla\ t \neq b \quad \wedge \quad t \neq a$

$\frac{dx}{dt} = (\frac{b}{b-t})' = \frac{b' \cdot (b-t) - b \cdot (b-t)'}{(b-t)^2} = \frac{0 - b \cdot (-1)}{(b-t)^2} = \frac{b}{(b-t)^2}$

$\frac{dy}{dt} = (\frac{a}{a-t})' = \frac{a' \cdot (a-t) - a \cdot (a-t)'}{(a-t)^2} = \frac{0 - a \cdot (-1)}{(a-t)^2} = \frac{a}{(a-t)^2}$

Zatem: $\frac{dy}{dx} = \frac{a}{(a-t)^2} \cdot \frac{(b-t)^2}{b} = \frac{a \cdot (b-t)^2}{b \cdot (a-t)^2} = \frac{a}{b} \cdot (\frac{b-t}{a-t})^2$ dla $b \neq 0$

7.16.

$x = \sqrt{t^2+1}, \qquad y = \frac{t-1}{\sqrt{t^2+1}}$

$\frac{dx}{dt} = (\sqrt{t^2+1})' = [(t^2+1)^{\frac{1}{2}}]' = \frac{1}{2} \cdot (t^2+1)^{\frac{1}{2}-1} \cdot (t^2+1)' = \frac{1}{2} \cdot \frac{1}{\sqrt{t^2+1}} \cdot 2t = \frac{t}{\sqrt{t^2+1}}$

$\frac{dy}{dt} = (\frac{t-1}{\sqrt{t^2+1}})' = \frac{(t-1)' \cdot \sqrt{t^2+1} - (t-1) \cdot (\sqrt{t^2+1})'}{(\sqrt{t^2+1})^2} = \{wykorzystujemy\, obliczenia\, wykonane\, wy\dot{z}ej\, dla\, (\sqrt{t^2+1})'\} =$

$= \frac{1 \cdot \sqrt{t^2+1} - (t-1) \cdot \frac{t}{\sqrt{t^2+1}}}{t^2+1} = \frac{\frac{(\sqrt{t^2+1})^2}{\sqrt{t^2+1}} - \frac{t \cdot (t-1)}{\sqrt{t^2+1}}}{t^2+1} = \frac{\frac{t^2+1-t^2+t}{\sqrt{t^2+1}}}{t^2+1} = \frac{t+1}{(t^2+1) \cdot \sqrt{t^2+1}}$

Zatem: $\frac{dy}{dx} = \frac{t+1}{(t^2+1) \cdot \sqrt{t^2+1}} \cdot \frac{\sqrt{t^2+1}}{t} = \frac{t+1}{t \cdot (t^2+1)}$

7.17.

$x = t^2 \qquad y = \frac{1}{3}t^3 - t$

$\frac{dx}{dt} = (t^2)' = 2t$

$\frac{dy}{dt} = (\frac{1}{3}t^3 - t)' = \frac{1}{3} \cdot 3 \cdot t^2 - 1 = t^2 - 1$

Zatem: $\frac{dy}{dx} = \frac{t^2-1}{2t}$

7.18.

$x = t^2 + 2t \qquad y = ln(t+1)$ dla $t > -1$

$\frac{dx}{dt} = (t^2 + 2t)' = 2t + 2 = 2(t+1)$

$\frac{dy}{dt} = [ln(t+1)]' = \frac{1}{t+1} \cdot (t+1)' = \frac{1}{t+1}$

Zatem: $\frac{dy}{dx} = \frac{1}{t+1} \cdot \frac{1}{2(t+1)} = \frac{1}{2(t+1)^2}$

7.19.

$x = \frac{2at}{1+t^2} \quad y = \frac{a(1-t^2)}{1+t^2}$

$\frac{dx}{dt} = \frac{(2at)' \cdot (1+t^2) - 2at(1+t^2)'}{(1+t^2)^2} = \frac{2a(1+t^2) - 2at \cdot 2t}{(1+t^2)^2} = \frac{2a + 2at^2 - 4at^2}{(1+t^2)^2} = \frac{2a - 2at^2}{(1+t^2)^2} = \frac{2a(1-t^2)}{(1+t^2)^2}$

$\frac{dy}{dt} = \frac{[a(1-t^2)]' \cdot (1+t^2) - a(1-t^2)(1+t^2)'}{(1+t^2)^2} = \frac{a(0-2t) \cdot (1+t^2) - (a-at^2)(0+2t)}{(1+t^2)^2} = \frac{-2at \cdot (1+t^2) - (a-at^2) \cdot 2t}{(1+t^2)^2} = \frac{-2at - 2at^3 - 2at + 2at^3}{(1+t^2)^2} =$

$= \frac{-4at}{(1+t^2)^2}$

Zatem: $\frac{dy}{dx} = \frac{-4at}{(1+t^2)^2} \cdot \frac{(1+t^2)^2}{2a(1-t^2)} = \frac{-2t}{1-t^2}$ dla $t \neq 1 \ \wedge \ t \neq -1$

7.20.

$x = a(t - sint), \quad y = a(1 - cost)$

$\frac{dx}{dt} = (at - asint)' = a - acost = a(1 - cost)$

$\frac{dy}{dt} = (a - acost)' = -a \cdot (-sint) = asint$

Zatem: $\frac{dy}{dx} = \frac{asint}{a(1-cost)} = \frac{sint}{1-cost} = (\frac{1-cost}{sint})^{-1} = (1)$

Skorzystamy teraz z wzoru: $tg\frac{\alpha}{2} = \frac{1-cos\alpha}{sin\alpha}$

$(1) = (tg\frac{t}{2})^{-1} = ctg\frac{t}{2}$, dla $(cost \neq 1 \wedge sint \neq 0) \Leftrightarrow (t \neq 2k\pi,\ k \in C \ \wedge \ t \neq k\pi,\ k \in C) \ \Leftrightarrow \ t \neq k\pi,\ k \in C$

7.21.

$x = acos^3t, \quad y = asin^3t$

$\frac{dx}{dt} = a[(cost)^3]' = a \cdot 3cos^2t \cdot (cost)' = 3acos^2t \cdot (-sint) = -3asint \cdot cos^2t$

$\frac{dy}{dt} = a[(sint)^3]' = a \cdot 3sin^2t \cdot (sint)' = 3asin^2t \cdot cost = 3asin^2t \cdot cost$

Zatem: $\frac{dy}{dx} = \frac{3asin^2t \cdot cost}{-3asint \cdot cos^2t} = -\frac{sint}{cost} = -tgt,$ dla $cost \neq 0$

7.22.

$x = cos2t, \quad y = sin^2 t$

$\frac{dx}{dt} = (cos2t)' = -sin2t \cdot (2t)' = -2sin2t$

$\frac{dy}{dt} = [(sint)^2]' = 2sint \cdot (sint)' = 2sint \cdot cost = (1)$

Korzystamy z wzoru: $sin2\alpha = 2sin\alpha \cdot cos\alpha$

$(1) = sin2t$

Zatem: $\frac{dy}{dx} = \frac{sin2t}{-2sin2t} = -\frac{1}{2}$, dla $sin2t \neq 0$

7.23.

$x = cos\varphi + \varphi sin\varphi, \quad y = sin\varphi - \varphi cos\varphi$

$\frac{dx}{d\varphi} = -sin\varphi + \varphi' \cdot sin\varphi + \varphi \cdot (sin\varphi)' = -sin\varphi + sin\varphi + \varphi cos\varphi = \varphi cos\varphi$

$\frac{dy}{d\varphi} = cos\varphi - (\varphi cos\varphi)' = cos\varphi - [\varphi' \cdot cos\varphi + \varphi \cdot (cos\varphi)'] = cos\varphi - cos\varphi - [\varphi \cdot (-sin\varphi)] = \varphi sin\varphi$

Zatem: $\frac{dy}{dx} = \frac{\varphi sin\varphi}{\varphi cos\varphi} = \frac{sin\varphi}{cos\varphi} = tg\varphi$

7.24.

$x = \alpha sint + sin(\alpha t), \quad y = \alpha cost + cos(\alpha t)$ przy $t = 0$

$\frac{dx}{dt} = \alpha cost + cos(\alpha t) \cdot (\alpha t)' = \alpha cost + \alpha cos(\alpha t)$

$\frac{dy}{dt} = -\alpha sint - sin(\alpha t) \cdot (\alpha t)' = -\alpha sint - \alpha sin(\alpha t)$

Zatem: $\frac{dy}{dx} = -\frac{\alpha sint + \alpha sin(\alpha t)}{\alpha cost + \alpha cos(\alpha t)} = -\frac{sint + sin(\alpha t)}{cost + cos(\alpha t)} = -\frac{2sin\frac{t+\alpha t}{2} \cdot cos\frac{t-\alpha t}{2}}{2cos\frac{t+\alpha t}{2} \cdot cos\frac{t-\alpha t}{2}} = -\frac{sin\frac{t+\alpha t}{2}}{cos\frac{t+\alpha t}{2}} = -tg(\frac{\alpha+1}{2} \cdot t)$

Dla $t = 0$ mamy: $\frac{dy}{dx}|_{t=0} = -tg0 = 0$

7.25.

$x = cost \cdot (cos2t)^{\frac{1}{2}}$, dla $y = sint \cdot (cos2t)^{\frac{1}{2}}$, $cos2t > 0$

$\frac{dx}{dt} = (cost)' \cdot (cos2t)^{\frac{1}{2}} + cost \cdot [(cos2t)^{\frac{1}{2}}]' = -sint \cdot (cos2t)^{\frac{1}{2}} + cost \cdot \frac{1}{2} \cdot (cos2t)^{\frac{1}{2}-1} \cdot (cos2t)' =$

$= -sint \cdot (cos2t)^{\frac{1}{2}} + cost \cdot \frac{1}{2} \cdot (cos2t)^{-\frac{1}{2}} \cdot (-sin2t) \cdot (2t)' = -sint \cdot (cos2t)^{\frac{1}{2}} - cost \cdot sin2t \cdot (cos2t)^{-\frac{1}{2}}$

$\frac{dy}{dt} = (sint)' \cdot (cos2t)^{\frac{1}{2}} + sint \cdot [(cos2t)^{\frac{1}{2}}]' = cost \cdot (cos2t)^{\frac{1}{2}} + sint \cdot \frac{1}{2} \cdot (cos2t)^{-\frac{1}{2}} \cdot (cos2t)' =$

$= cost \cdot (cos2t)^{\frac{1}{2}} + sint \cdot \frac{1}{2} \cdot (cost2t)^{-\frac{1}{2}} \cdot (-sin2t) \cdot (2t)' = cost \cdot (cos2t)^{\frac{1}{2}} - sint \cdot sin2t \cdot (cos2t)^{-\frac{1}{2}}$

Zatem: $\frac{dy}{dx} = \frac{cost \cdot (cos2t)^{\frac{1}{2}} - sint \cdot sin2t \cdot (cos2t)^{-\frac{1}{2}}}{-sint \cdot (cos2t)^{\frac{1}{2}} - cost \cdot sin2t \cdot (cos2t)^{-\frac{1}{2}}} = \frac{cost \cdot cos2t - sint \cdot sin2t \cdot 1}{-sint \cdot cos2t - cost \cdot sin2t \cdot 1} = -\frac{cos(t+2t)}{sin(t+2t)} = -\frac{cost3t}{sin3t} = -ctg3t$
dla $sin3t \neq 0$

Skorzystaliśmy tu ze wzorów:
$sin(\alpha + \beta) = sin\alpha \cdot cos\beta + cos\alpha \cdot sin\beta$ oraz $cos(\alpha + \beta) = cos\alpha \cdot cos\beta - sin\alpha \cdot sin\beta$

7.26.

$x = \frac{cos^3t}{\sqrt{cos2t}}$ $y = \frac{sin^3t}{\sqrt{cos2t}}$ dla $cos2t > 0$

$\frac{dx}{dt} = \frac{(cos^3t)' \cdot \sqrt{cos2t} - cos^3t \cdot [(cos2t)^{\frac{1}{2}}]'}{(\sqrt{cos2t})^2} = \frac{3cos^2t \cdot (cost)' \cdot (cos2t)^{\frac{1}{2}} - cos^3t \cdot \frac{1}{2} \cdot (cos2t)^{-\frac{1}{2}} \cdot (cos2t)'}{cos2t} =$

$= \frac{3cos^2t \cdot (-sint) \cdot (cos2t)^{\frac{1}{2}} - cos^3t \cdot \frac{1}{2} \cdot (cos2t)^{-\frac{1}{2}} \cdot (-sin2t) \cdot (2t)'}{cos2t} = \frac{-3cos^2t \cdot sint \cdot (cos2t)^{\frac{1}{2}} + cos^3t \cdot \frac{1}{2} \cdot (cos2t)^{-\frac{1}{2}} \cdot sin2t \cdot 2}{cos2t} =$

$= \frac{sin2t \cdot cos^3t \cdot (cos2t)^{-\frac{1}{2}} - 3 \cdot sint \cdot cos^2t \cdot (cos2t)^{\frac{1}{2}}}{cos2t}$

$\frac{dy}{dt} = \frac{(sin^3t)' \cdot \sqrt{cos2t} - sin^3t \cdot [(cos2t)^{\frac{1}{2}}]'}{(\sqrt{cos2t})^2} = \frac{3sin^2t \cdot (sint)' \cdot (cos2t)^{\frac{1}{2}} - sin^3t \cdot \frac{1}{2} \cdot (cos2t)^{-\frac{1}{2}} \cdot (cos2t)'}{cos2t} =$

$= \frac{3sin^2t \cdot cost \cdot (cos2t)^{\frac{1}{2}} - sin^3t \cdot \frac{1}{2} \cdot (cos2t)^{-\frac{1}{2}} \cdot (-sin2t) \cdot (2t)'}{cos2t} = \frac{3sin^2t \cdot cost \cdot (cos2t)^{\frac{1}{2}} + sin^3t \cdot \frac{1}{2} \cdot (cos2t)^{-\frac{1}{2}} \cdot sin2t \cdot 2}{cos2t} =$

$= \frac{3sin^2t \cdot cost \cdot (cos2t)^{\frac{1}{2}} + sin^3t \cdot (cos2t)^{-\frac{1}{2}} \cdot sin2t}{cos2t}$

Zatem $\frac{dy}{dx} = \frac{3sin^2t \cdot cost \cdot (cos2t)^{\frac{1}{2}} + sin^3t \cdot (cos2t)^{-\frac{1}{2}} \cdot sin2t}{cos2t} \cdot \frac{cos2t}{sin2t \cdot cos^3t \cdot (cos2t)^{-\frac{1}{2}} - 3 \cdot sint \cdot cos^2t \cdot (cos2t)^{\frac{1}{2}}} =$

$= \frac{3sin^2t \cdot cost \cdot (cos2t)^{\frac{1}{2}} + sin^3t \cdot (cos2t)^{-\frac{1}{2}} \cdot sin2t}{sin2t \cdot cos^3t \cdot (cos2t)^{-\frac{1}{2}} - 3 \cdot sint \cdot cos^2t \cdot (cos2t)^{\frac{1}{2}}} = \frac{3sin^2t \cdot cost \cdot cos2t + sin^3t \cdot 1 \cdot sin2t}{sin2t \cdot cos^3t \cdot 1 - 3 \cdot sint \cdot cos^2t \cdot cos2t} = \frac{sin^2t \cdot (3 \cdot cost \cdot cos2t + sint \cdot sin2t)}{cos^2t \cdot (sin2t \cdot cost - 3 \cdot sint \cdot cos2t)} =$

$= \frac{sin^2t \cdot (3 \cdot cost \cdot cos2t + sint \cdot sin2t)}{cos^2t \cdot (sin2t \cdot cost - 3 \cdot sint \cdot cos2t)} = \frac{L}{M}$

Zastosujmy do $\frac{L}{M}$ wzory:

$sin2\alpha = 2sin\alpha \cdot cos\alpha$

$cos2\alpha = cos^2\alpha - sin^2\alpha$

$sin3\alpha = sin\alpha \cdot (3cos^2\alpha - sin^2\alpha)$

$cos3\alpha = cos\alpha \cdot (cos^2\alpha - 3sin^2\alpha)$

$L = sin^2t \cdot [3 \cdot cost \cdot (cos^2t - sin^2t) + sint \cdot (2sint \cdot cost)] =$

$= sin^2t \cdot (3cos^3t - 3sin^2t \cdot cost + 2sin^2t \cdot cost) = sin^2t \cdot (3cos^3t - sin^2t \cdot cost) =$

$= sin^2t \cdot cost \cdot (3cos^2t - sin^2t) = sint \cdot cost \cdot sin3t$

$M = cos^2t \cdot [(2 \cdot sint \cdot cost) \cdot cost - 3 \cdot sint \cdot (cos^2t - sin^2t)] =$

$= cos^2t \cdot (2 \cdot sint \cdot cos^2t - 3 \cdot sint \cdot cos^2t + 3sin^3t) = cos^2t \cdot (3sin^3t - sint \cdot cos^2t) =$

$= -cos^2t \cdot sint \cdot (cos^2t - 3sin^2t) = -sint \cdot cost \cdot cos3t$

I ostatecznie: $\frac{L}{M} = \frac{sint \cdot cost \cdot sin3t}{-sint \cdot cost \cdot cos3t} = -\frac{sin3t}{cos3t} = -tg3t$

7.27.
$x = ln(tg\frac{1}{2}t) + cost - sint$ dla $tg\frac{1}{2}t > 0$
$y = sint + cost$

$\frac{dx}{dt} = [ln(tg\frac{1}{2}t)]' + (cost)' - (sint)' = \frac{1}{tg\frac{1}{2}t} \cdot (tg\frac{1}{2}t)' - sint - cost = \frac{1}{tg\frac{1}{2}t} \cdot \frac{1}{cos^2\frac{1}{2}t} \cdot (\frac{1}{2}t)' - sint - cost =$

$= \frac{cos\frac{1}{2}t}{sin\frac{1}{2}t} \cdot \frac{1}{cos^2\frac{1}{2}t} \cdot \frac{1}{2} - sint - cost = \frac{1}{2sin\frac{1}{2}t \cdot cos\frac{1}{2}t} - sint - cost = \frac{1}{sin(2 \cdot \frac{1}{2}t)} - sint - cost =$

$= \frac{1}{sint} - sint - cost = \frac{1 - sin^2t - sint \cdot cost}{sint} = \frac{cos^2t - sint \cdot cost}{sint} = \frac{cost \cdot (cost - sint)}{sint}$

$\frac{dy}{dt} = (sint)' + (cost)' = cost - sint$

Zatem: $\frac{dy}{dx} = (cost - sint) \cdot \frac{sint}{cost \cdot (cost - sint)} = \frac{sint}{cost} = tgt$ dla $cost \neq 0 \ \wedge \ cost \neq sint$

7.28.

$x = alnt, \qquad y = \frac{a}{2} \cdot (t + \frac{1}{t})$ dla $t > 0$

$\frac{dx}{dt} = a \cdot (lnt)' = a \cdot \frac{1}{t}$

$\frac{dy}{dt} = \frac{a}{2} \cdot (t + \frac{1}{t})' = \frac{a}{2} \cdot [t' + (t^{-1})'] = \frac{a}{2} \cdot (1 - 1 \cdot t^{-2}) = \frac{a}{2} \cdot (1 - \frac{1}{t^2})$

Zatem: $\frac{dy}{dx} = \frac{\frac{1}{2} \cdot a \cdot (1 - \frac{1}{t^2})}{\frac{a}{t}} = \frac{\frac{1}{2} \cdot a \cdot t \cdot (1 - \frac{1}{t^2})}{a} = \frac{1}{2} \cdot t \cdot (1 - \frac{1}{t^2}) = \frac{1}{2} \cdot (t - \frac{1}{t})$

7.29.

$x = \frac{at}{1+t^3}, \qquad y = \frac{at^2}{1+t^3}$ dla $t - 1$

$\frac{dx}{dt} = \frac{(at)' \cdot (1+t^3) - at \cdot (1+t^3)'}{(1+t^3)^2} = \frac{a \cdot (1+t^3) - at \cdot (0+3t^2)}{(1+t^3)^2} = \frac{a \cdot (1+t^3-3t^3)}{(1+t^3)^2} = \frac{a \cdot (1-2t^3)}{(1+t^3)^2}$

$\frac{dy}{dt} = \frac{(at^2)' \cdot (1+t^3) - at^2 \cdot (1+t^3)'}{(1+t^3)^2} = \frac{2at \cdot (1+t^3) - at^2 \cdot (0+3t^2)}{(1+t^3)^2} = \frac{2at \cdot (1+t^3) - at^2 \cdot 3t^2}{(1+t^3)^2} = \frac{at \cdot (2+2t^3-3t^3)}{(1+t^3)^2} = \frac{at \cdot (2-t^3)}{(1+t^3)^2}$

Zatem: $\frac{dy}{dx} = \frac{at \cdot (2-t^3)}{(1+t^3)^2} \cdot \frac{(1+t^3)^2}{a \cdot (1-2t^3)} = \frac{t \cdot (2-t^3)}{1-2t^3}$ dla $t^3 \neq \frac{1}{2}$

7.30.

$x = (R+r)cost - Rcos(\frac{R+r}{R} \cdot t), \qquad y = (R+r)sint - Rsin(\frac{R+r}{R} \cdot t)$

$\frac{dx}{dt} = (R+r)(cost)' - R \cdot [cos(\frac{R+r}{R} \cdot t)]' = -(R+r)sint + R \cdot sin(\frac{R+r}{R} \cdot t) \cdot (\frac{R+r}{R} \cdot t)' =$

$= -(R+r)sint + R \cdot \frac{R+r}{R} \cdot sin(\frac{R+r}{R} \cdot t) = (R+r) \cdot [sin(\frac{R+r}{R} \cdot t) - sint]$

$\frac{dy}{dt} = (R+r)(sint)' - R[sin(\frac{R+r}{R} \cdot t)]' = (R+r)cost - Rcos(\frac{R+r}{R} \cdot t) \cdot (\frac{R+r}{R} \cdot t)' =$

$= (R+r)cost - R \cdot \frac{R+r}{R} \cdot cos(\frac{R+r}{R} \cdot t) = (R+r) \cdot [cost - cos(\frac{R+r}{R} \cdot t)]$

Zatem: $\frac{dy}{dx} = \frac{(R+r) \cdot [cost - cos(\frac{R+r}{R} \cdot t)]}{(R+r) \cdot [sin(\frac{R+r}{R} \cdot t) - sint]} = \frac{cost - cos(\frac{R+r}{R} \cdot t)}{sin(\frac{R+r}{R} \cdot t) - sint} = \frac{L}{M}$

Przekształćmy teraz licznik i mianownik powyższego wyrażenia korzystając z wzorów:
$sin\alpha - sin\beta = 2cos\frac{\alpha+\beta}{2} \cdot sin\frac{\alpha-\beta}{2}$
$cos\alpha - cos\beta = -2sin\frac{\alpha+\beta}{2} \cdot sin\frac{\alpha-\beta}{2}$

Mamy więc: $L = cost - cos(\frac{R+r}{R} \cdot t) = -2sin[\frac{1}{2} \cdot (t + \frac{R+r}{R}t)] \cdot sin[\frac{1}{2} \cdot (t - \frac{R+r}{R}t)] =$

$= -2sin[\frac{1}{2}t \cdot (1 + \frac{R+r}{R})] \cdot sin[\frac{1}{2}t \cdot (1 - \frac{R+r}{R})] = -2sin(\frac{1}{2}t \cdot \frac{R+R+r}{R}) \cdot sin(\frac{1}{2}t \cdot \frac{R-R+r}{R}) =$

$= -2sin(\frac{1}{2}t \cdot \frac{2R+r}{R}) \cdot sin(\frac{1}{2}t \cdot \frac{r}{R})$

oraz:

$M = sin(\frac{R+r}{R} \cdot t) - sint = 2cos[\frac{1}{2} \cdot (\frac{R+r}{R}t + t)] \cdot sin[\frac{1}{2} \cdot (\frac{R+r}{R}t - t)] =$

$= 2cos[\frac{1}{2}t \cdot (\frac{R+r}{R} + 1)] \cdot sin[\frac{1}{2}t \cdot (\frac{R+r}{R} - 1)] = 2cos[\frac{1}{2}t \cdot (\frac{R+r+R}{R})] \cdot sin[\frac{1}{2}t \cdot (\frac{R+r-R}{R})] =$

$= 2cos(\frac{1}{2}t \cdot \frac{2R+r}{R}) \cdot sin(\frac{1}{2}t \cdot \frac{r}{R})$

Czyli ostatecznie: $\frac{dy}{dx} = \frac{L}{M} = \frac{-2sin(\frac{1}{2}t \cdot \frac{2R+r}{R}] \cdot sin(\frac{1}{2}t \cdot \frac{r}{R})}{2cos(\frac{1}{2}t \cdot \frac{2R+r}{R}) \cdot sin(\frac{1}{2}t \cdot \frac{r}{R})} = -\frac{sin(\frac{1}{2}t \cdot \frac{2R+r}{R}]}{cos(\frac{1}{2}t \cdot \frac{2R+r}{R})} = -tg(\frac{1}{2}t \cdot \frac{2R+r}{R}) = -tg(\frac{2R+r}{2R}t),$

dla $cos(\frac{1}{2}t \cdot \frac{2R+r}{R}) \neq 0 \quad \wedge \quad sin(\frac{1}{2}t \cdot \frac{r}{R}) \neq 0$

7.31.

$x = 1 + e^{a\varphi} \qquad\qquad y = a\varphi + e^{-a\varphi}$

$\frac{dx}{d\varphi} = 1' + (e^{a\varphi})' = 0 + e^{a\varphi} \cdot (a\varphi)' = ae^{a\varphi}$

$\frac{dy}{d\varphi} = (a\varphi)' + (e^{-a\varphi})' = a + e^{-a\varphi} \cdot (-a\varphi)' = a - ae^{-a\varphi} = a(1 - e^{-a\varphi})$

Zatem: $\frac{dy}{dx} = \frac{a(1-e^{-a\varphi})}{ae^{a\varphi}} = \frac{1-e^{-a\varphi}}{e^{a\varphi}} = \frac{1}{e^{a\varphi}} - \frac{e^{-a\varphi}}{e^{a\varphi}} = e^{-a\varphi} - e^{-a\varphi} \cdot e^{-a\varphi} = e^{-a\varphi} - e^{-2a\varphi}$

7.32.

$x = e^{at} \qquad\qquad y = e^{-at}$

$\frac{dx}{dt} = (e^{at})' = e^{at} \cdot (at)' = ae^{at}$

$\frac{dy}{dt} = (e^{-at})' = e^{-at} \cdot (-at)' = -ae^{-at}$

Zatem: $\frac{dy}{dx} = \frac{-ae^{-at}}{ae^{at}} = -e^{-at} \cdot e^{-at} = -e^{-2at}$

7.33.

Uwaga! Żeby rozwiązanie zgadzało się z tym w odpowiedziach, wzór dla wartości y musi być podniesiony do potęgi trzeciej a nie drugiej jak to jest w treści zadania.

$x = (e^{at} - 1)^2 \qquad y = (e^{at} - 1)^3$

$\frac{dx}{dt} = [(e^{at} - 1)^2]' = 2 \cdot (e^{at} - 1) \cdot (e^{at} - 1)' = 2 \cdot (e^{at} - 1) \cdot (e^{at})' = 2 \cdot (e^{at} - 1) \cdot e^{at} \cdot (at)' =$

$= 2a \cdot (e^{at} - 1) \cdot e^{at}$

$\frac{dy}{dt} = [(e^{at} - 1)^3]' = 3 \cdot (e^{at} - 1)^2 \cdot (e^{at} - 1)' = 3 \cdot (e^{at} - 1)^2 \cdot e^{at} \cdot (at)' = 3a \cdot (e^{at} - 1)^2 \cdot e^{at}$

Zatem: $\frac{dy}{dx} = \frac{3a \cdot (e^{at}-1)^2 \cdot e^{at}}{2a \cdot (e^{at}-1) \cdot e^{at}} = \frac{3}{2} \cdot (e^{at} - 1)$

7.34.

$x = arccos\frac{1}{\sqrt{1+t^2}} \qquad y = arcsin\frac{t}{\sqrt{1+t^2}}$

$\frac{dx}{dt} = (arccos\frac{1}{\sqrt{1+t^2}})' = \frac{-1}{\sqrt{1-(\frac{1}{\sqrt{1+t^2}})^2}} \cdot [(1+t^2)^{-\frac{1}{2}}]' = -\frac{1}{\sqrt{1-\frac{1}{1+t^2}}} \cdot (-\frac{1}{2}) \cdot (1+t^2)^{-\frac{3}{2}} \cdot (1+t^2)' =$

$= -\frac{1}{\sqrt{\frac{1+t^2-1}{1+t^2}}} \cdot (-\frac{1}{2}) \cdot (1+t^2)^{-\frac{3}{2}} \cdot 2t = \frac{1}{\sqrt{\frac{1+t^2-1}{1+t^2}}} \cdot (1+t^2)^{-\frac{3}{2}} \cdot t = \frac{t(1+t^2)^{-\frac{3}{2}}}{\sqrt{\frac{t^2}{1+t^2}}} = \frac{t(1+t^2)^{-\frac{3}{2}} \cdot (1+t^2)^{\frac{1}{2}}}{\sqrt{t^2}} =$

$= \frac{t(1+t^2)^{-1}}{|t|} = \frac{t}{|t|} \cdot (1+t^2)^{-1}$

$\frac{dy}{dt} = (arcsin\frac{t}{\sqrt{1+t^2}})' = \frac{1}{\sqrt{1-(\frac{t}{\sqrt{1+t^2}})^2}} \cdot (\frac{t}{\sqrt{1+t^2}})' = \frac{1}{\sqrt{1-\frac{t^2}{1+t^2}}} \cdot [\frac{t}{(1+t^2)^{\frac{1}{2}}}]' = \frac{1}{\sqrt{\frac{1+t^2-t^2}{1+t^2}}} \cdot \frac{t' \cdot (1+t^2)^{\frac{1}{2}} - t \cdot [(1+t^2)^{\frac{1}{2}}]'}{[(1+t^2)^{\frac{1}{2}}]^2} =$

$= \frac{1}{\sqrt{\frac{1}{1+t^2}}} \cdot \frac{(1+t^2)^{\frac{1}{2}} - t \cdot \frac{1}{2} \cdot (1+t^2)^{-\frac{1}{2}} \cdot (1+t^2)'}{1+t^2} = (1+t^2)^{\frac{1}{2}} \cdot \frac{(1+t^2)^{\frac{1}{2}} - \frac{1}{2} t \cdot (1+t^2)^{-\frac{1}{2}} \cdot 2t}{1+t^2} =$

$= (1+t^2)^{\frac{1}{2}} \cdot [(1+t^2)^{-\frac{1}{2}} - t^2 \cdot (1+t^2)^{-\frac{3}{2}}] = 1 - t^2 \cdot (1+t^2)^{-1}$

Zatem: $\frac{dy}{dx} = \frac{1-t^2 \cdot (1+t^2)^{-1}}{\frac{t}{|t|} \cdot (1+t^2)^{-1}} = \frac{(1+t^2)-t^2 \cdot (1+t^2)^{-1} \cdot (1+t^2)}{\frac{t}{|t|} \cdot (1+t^2)^{-1} \cdot (1+t^2)} = \frac{1+t^2-t^2 \cdot 1}{\frac{t}{|t|} \cdot 1} = \frac{|t| \cdot 1}{t} = \frac{|t|}{t} = \begin{cases} 1 & dla \;\; t > 0 \\ -1 & dla \;\; t < 0 \end{cases}$

dla $t \neq 0$